11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

FRAME INVARIANT AND ENTROPIC SECOND ORDER CELL-CENTERED ALE SCHEMES

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Key words: *Compressible hydrodynamics, Cell-centered schemes, Galilean invariance, ALE methods, Local Convex Hull Preservation (LCHP).*

We describe the extension of previous works [5], dedicated to the frame invariance and the maximum principle preserving of cell-centered ALE schemes.

We propose a spatial second-order extension for cell-centered ALE schemes solving compressible Euler equations. This extension aims to be linear preserving, Galilean invariant, and extrema preserving on intensive variables (velocity and specific internal energy). For second-order extension, to design a limiter preserving Galilean invariance is challenging for vectors and tensors, because a component by component limitation breaks this property. Recent works devoted to this topic are for instance [7, 8]. Another approach has been developed by Luttwak and Falkovitz in [6].

This work is based on the latter method called VIP for Vector Image Polygon. It consists in enforcing the reconstructed velocities to remain into the convex hull induced by the neighbouring velocities. We applied this method in the context of a Lagrange+Remap ALE algorithm. The Lagrangian scheme is Glace [2] or Eucclhyd [7]. Remap is performed using the Benson-like algorithm described in [4].

The originality of this work is to preserve the extrema of the momentum during the remapping phase, but also of the velocity. In order to enforce this property, we adapt the work of [3, 1] to our vector limiter. The main idea is to perform a homothetic transformation of the convex hull after centering on the first-order velocity value. We call this procedure the Local Convex Hull Preservation, which is the vectorial extension of the maximum principle. Moreover, using this idea, we are able to do the link with some classical limiters for scalar fields.

In the present work, we enforce, in addition, some desirable features to the Lagrangian step of the ALE algorithm as, for instance, the entropy increase. We use the same techniques of a posteriori limitation, relying on the fact that the first-order Lagrangian scheme is entropic. Similar idea has been developed in the Eulerian framework in [3].

This gives us a fully Galilean invariant and entropic second-order ALE method for the compressible Euler equations (assuming rezoning is Galilean invariant). This method is assessed on several classical test problems.

REFERENCES

[1] S. Clain, S. Diot and R. Loubère. A high-order finite volume method for systems of conservation laws-Multi-Dimensional Optimal Order Detection (MOOD). *J. Comput. Phys.*, Vol. **230**, 4028-4050, 2011.

[2] B. Després and C. Mazeran. Lagrangian gas dynamics in two dimensions and Lagrangian systems. *Arch. Rational Mech. Anal.*, Vol. **178**, 327-372, 2005.

[3] V. Desveaux. Contribution à l'approximation numérique des systèmes hyperboliques. *PhD Thesis*, Nantes university, 2013.

[4] Ph. Hoch. An arbitrary Lagrangian-Eulerian strategy to solve compressible flows. *Technical Report*, HAL:hal00366858, CEA, 2009.

[5] Ph. Hoch and E. Labourasse. Local convex-Hull preserving second-order extension for cell-centered ALE schemes. *Multimat 2013*, San Francisco, 2013.

[6] G. Luttwak and F. Falkovitz. Slope limiting for vectors: a novel vector limiting algorithm. *Numerical Methods in Fluids*, Vol. **65**, 2011.

[7] P.H. Maire. Contribution to the numerical modeling of inertial confinement fusion. *Technical Report, CEAR6260*, CEA/DAM, 2009.

[8] S. K. Sambasivan, M. J. Shashkov and D. E. Burton. Exploration of new limiter schemes for stress tensors in Lagrangian and ALE hydrocodes, *Computer and Fluids*, Vol. **83**, 98–114, 2013.