

MULTIFIELD FORMULATION OF PLASTICITY

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The development of lightweight constructions and functionally optimized mass products follows different strategies. On the one hand side new materials like composite structures as well as enhanced alloys are created, on the other hand innovative production technologies are investigated. An integrated thermomechanical forming process can be considered as one example. Therein a steel shaft is heated inductively creating a heterogeneous temperature field, forged thermomechanically due to the contact with the forming die and in the last step a local cooling using a high pressured air stream is carried out. Because of the resulting distribution of steel phases the material properties can be application oriented adjusted following this production sequence, cf. [1].

In order to predict these phase distributions properly higher order accurate numerical simulation methods are necessary for the individual manufacturing stages. Considering the application of adequate time integration schemes the forging process, characterized by plasticity effects, embodies an interesting point of departure since reliable time integration schemes are often only employed considering special cases. Hence, a conventional material model of the VON MISES type is employed to derive a consistent variational formulation of plasticity exploiting the principle of JOURDAIN, cf. [2, 3, 4]. Furthermore, a dissipation potential is assumed, cf. [5, 6]. Throughout this procedure the measurements concerning plasticity effects are no longer calculated on GAUSS point level, thus the classical radial return map algorithm becomes superfluous leading to a single level NEWTON iteration. As a consequence an effective active set strategy is developed to judge whether elastic or plastic phenomena prevail at distinct element nodes. Additionally, higher order accurate GALERKIN time discretization methods and their inherit error estimators are applied, cf. [7, 8].

REFERENCES

- [1] K. Steinhoff, U. Weidig and N. Saba. *Investigation of Plastic Forming Under the Influence of Locally and Temporally Variable Temperature and Stress States*. In: *Functionally Graded Materials in Industrial Mass Production*, Verlag Wissenschaftliche Scripten, 2009.
- [2] K. Ammar, B. Appolaire, G. Cailletaud, F. Feyel and S. Forest. Finite element formulation of a phase field model based on the concept of generalized stresses. *Computer Materials Science*, Vol. **45**, 800–805, 2008.
- [3] P. Germain. The method of virtual power in continuum mechanics. Part 2: Microstructure. *SIAM Journal on Applied Mathematics*, Vol. **25**, 556–576, 1973.
- [4] J. C. Simo and C. Miehe. Associative coupled thermoplasticity at finite strains: Formulation, numerical analysis and implementation. *Computer Methods in Applied Mechanics and Engineering*, Vol. **98**, 41–104, 1992.
- [5] K. Hackl. Generalized standard media and variational principles in classical and finite strain elastoplasticity. *Journal of the Mechanics and Physics of Solids*, Vol. **45**, 667–688, 1997.
- [6] C. Miehe. A multi-field incremental variational framework for gradient-extended standard dissipative solids. *Journal of Mechanics and Physics of Solids*, Vo. **59**, 898–923, 2011.
- [7] S. Carstens and D. Kuhl. Higher-order accurate implicit time integration schemes for transport problems. *Archive of Applied Mechanics*, Vol. **82**, 1007–1039, 2012.
- [8] T. Gleim and D. Kuhl. Higher order accurate discontinuous and continuous p-Galerkin methods for linear elastodynamics. *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. **93**, 177–194, 2013.