

# WELL-BALANCED $R$ -ADAPTIVE AND MOVING MESH SPACE-TIME DISCONTINUOUS GALERKIN METHOD FOR THE SHALLOW WATER EQUATIONS

S. Rhebergen

University of Oxford, Mathematical Institute, Andrew Wiles Building, Radcliffe Observatory  
Quarter, Woodstock Road, Oxford OX2 6GG, United Kingdom,  
sander.rhebergen@maths.ox.ac.uk

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To accurately solve the shallow water equations on moving and deforming meshes is non-trivial. The main problem one faces on moving meshes is how to maintain a conservative numerical discretization and it has been shown (e.g. Lesoinne and Farhat [1]) that the so-called Geometric Conservation Law (GCL) is essential on moving meshes for the time-accuracy of the solution.

The space-time Discontinuous Galerkin (DG) method, in which space- and time are discretized simultaneously on a space-time domain, automatically satisfies the GCL and has proven to be an excellent method to achieve accurate solutions on moving meshes. In this talk I will discuss how to obtain a well-balanced space-time DG method *even* on moving meshes and deforming domains [2]. Because of this well-balancedness property on moving meshes, it provides a natural framework in which to incorporate  $r$ -adaptivity in which nodes of the mesh are moved to areas of interest, therefore locally increasing the resolution without increasing the degrees of freedom. I will show numerical simulations in which I test well-balancedness on moving meshes and deforming domains and the use of  $r$ -adaptivity.

## REFERENCES

- [1] M. Lesoinne and C. Farhat. Geometric conservation laws for flow problems with moving boundaries and deformable meshes, and their impact on aeroelastic computations. *Comput. Methods Appl. Mech. Engrg.*, Vol. **134**, 71–90, 1996.
- [2] S. Rhebergen, Well-balanced  $r$ -adaptive and moving mesh space-time discontinuous Galerkin method for the shallow water equations, submitted 2013. Currently available as technical report on Oxford eprints: <http://eprints.maths.ox.ac.uk/1757/>