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NUMERICAL SOLUTION OF LINEAR EIGENPROBLEMS CONTAINING DERIVATIVES OF THE TANGENT STIFFNESS MATRIX WITH RESPECT TO THE LOAD PARAMETER

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To visualize the concept of energy-based categorization of buckling problems, the so-called buckling sphere was recently proposed [1]. The two spherical coordinates, which describe the position of an arbitrary point on an octant of this sphere, allow computation of the ratio of the bending energy to the total strain energy and of the one of the rotation-dependent part of the membrane energy to the total membrane energy for arbitrary values of a dimensionless parameter λ in the prebuckling regime and at loss of stability by bifurcation buckling or snap-through.

The so-called consistently linearized eigenproblem, which was firstly mentioned in [2], plays a fundamental role in the implementation of this concept. Its mathematical expression reads as

$$(\widetilde{\boldsymbol{K}}_T + (\lambda_j^* - \lambda) \cdot \dot{\widetilde{\boldsymbol{K}}}_T) \cdot \boldsymbol{v}_j^* = \boldsymbol{0}, \quad j = 1, 2, 3, ..., N, \tag{1}$$

where $\widetilde{\boldsymbol{K}}_T$ denotes the tangent stiffness matrix of a structure, in the frame of the Finite Element Method (FEM), evaluated along the primary path,

$$\dot{\widetilde{\boldsymbol{K}}}_T := \frac{d\widetilde{\boldsymbol{K}}_T}{d\lambda}, \tag{2}$$

and $(\lambda_j^* - \lambda, \boldsymbol{v}_j^*)$ is the *j-th* eigenpair, with $\boldsymbol{v}_j^* \cdot \boldsymbol{v}_j^* = 1$.

Computation of the azimuth angle φ requires knowledge of $\dot{\widetilde{\boldsymbol{K}}}_T$ and $\boldsymbol{v}_1^*$ [1]. $\dot{\widetilde{\boldsymbol{K}}}_T$ is needed for computation of the eigenpair $(\lambda_1^* - \lambda, \boldsymbol{v}_1^*)$ (see (1)). Computation of the zenith angle θ

requires knowledge of α_1 and $\dot{\lambda}_1^*$, with α_1 following from the solution of the eigenproblem [1]

$$[\frac{\alpha_1}{\lambda_1^* - \lambda}\dot{\widetilde{\boldsymbol{K}}}_T + \ddot{\widetilde{\boldsymbol{K}}}_T] \cdot \hat{\boldsymbol{v}} = \boldsymbol{0}, \tag{3}$$

where

$$\ddot{\widetilde{\boldsymbol{K}}}_T := \frac{d\dot{\widetilde{\boldsymbol{K}}}_T}{d\lambda}. \tag{4}$$

$\dot{\lambda}_1^*$ is obtained from the following relation [2]:

$$\dot{\lambda}_1^* = -(\lambda_1^* - \lambda)\frac{\boldsymbol{v}_1^* \cdot \ddot{\widetilde{\boldsymbol{K}}}_T \cdot \boldsymbol{v}_1^*}{\boldsymbol{v}_1^* \cdot \dot{\widetilde{\boldsymbol{K}}}_T \cdot \boldsymbol{v}_1^*}. \tag{5}$$

The main difficulties encountered in the numerical solution of (1) and (3) are

(I) an effective numerical computation of $\dot{\widetilde{\boldsymbol{K}}}_T$ and $\ddot{\widetilde{\boldsymbol{K}}}_T$,

(II) a comparative evaluation of the relative merits of one of the two different approaches of computation of these two matrices, and

(III) the assessment of the quality of the solutions for λ_1^* and α_1, based on these two approaches.

The main purpose of this paper is to describe how these difficulties are tackled. A parabolic arch subjected to a uniformly distributed load serves as an example for the numerical verification of the proposed concept.

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