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# A reduced basis technique with the co-rotational kinematics for nonlinear buckling analysis of structures

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Buckling of thin-walled structures is an inherently nonlinear phenomena. Thus, the analysis of nonlinear response of structures is important for determining their load carrying capability. Nowadays, with the increasing computational power of modern computers nonlinear finite element analysis is becoming the standard technique used to obtain the nonlinear response of complex structures, however, the repeated analyses that are needed in the design phase are still computationally intensive. For this reason, reduced order techniques that reduce the problem size are attractive whenever repetitive analyses are required.

In this contribution, a new approach, the Koiter-Newton[1,2] method, is presented for the numerical solution of a class of elastic nonlinear structural analysis problems. The method combines ideas from Koiter's initial post-buckling analysis and Newton arc-length methods to obtain an algorithm that is accurate over the entire equilibrium path of structures and efficient in the presence of buckling and/or imperfection sensitivity. The basic premise behind the proposed approach is the use of Koiter's asymptotic expansion from the beginning rather than using it only at the bifurcation point in contrast to the traditional Koiter approaches. The proposed approach is performed in a step by step manner to trace the entire equilibrium path. In every expansion step, the method combines a prediction step using a nonlinear reduced order model based on Koiter's initial post-buckling expansion and a Newton correction procedure. This nonlinear prediction provided by the reduced order model allows the use of fairly large step sizes and thus shows a superior efficiency in terms of numerical effort compared to linear predictors used by the classical Newton-Raphson method, cf Fig 1.

The proposed technique requires derivatives of the element load vectors with respect to the degrees of freedom up to the third order. This is two orders more than what is traditionally needed for Newton's method. To facilitate differentiation, nonlinear elements based on the element independent co-rotational frame are applied in the Koiter-Newton analysis. Automatic differentiation is used to find the derivatives of the co-rotational frame with respect to element degrees of freedom. In this way, full nonlinear kinematics are taken into account when constructing the reduced order model.

We illustrate the good performance of the method with several numerical examples of beam and shell models. We further demonstrate that for the Scordelis-Lo shell benchmark from the undeformed state only three steps are necessary to trace the complete equilibrium path, cf Fig 2. Furthermore, it is shown by comparison with results obtained from ABAQUS which adopts a full nonlinear analysis that co-rotational kinematics and von Karman kinematics as used in our Koiter-Newton approach further increases the efficiency in terms of accuracy and computational effort.

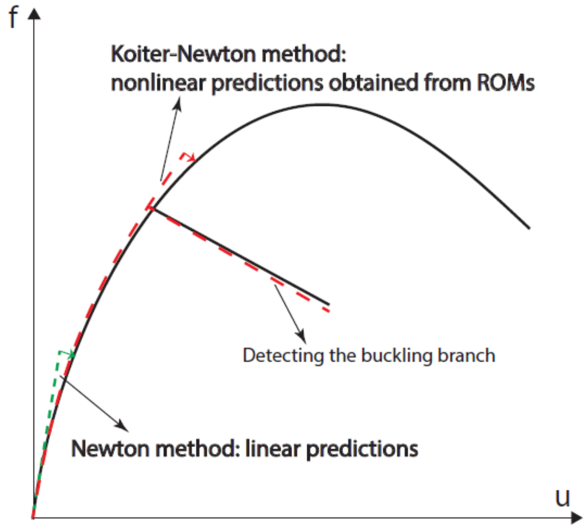


*Fig. 1 Path-following strategy of the Koiter-Newton approach, compared with Newton methods*

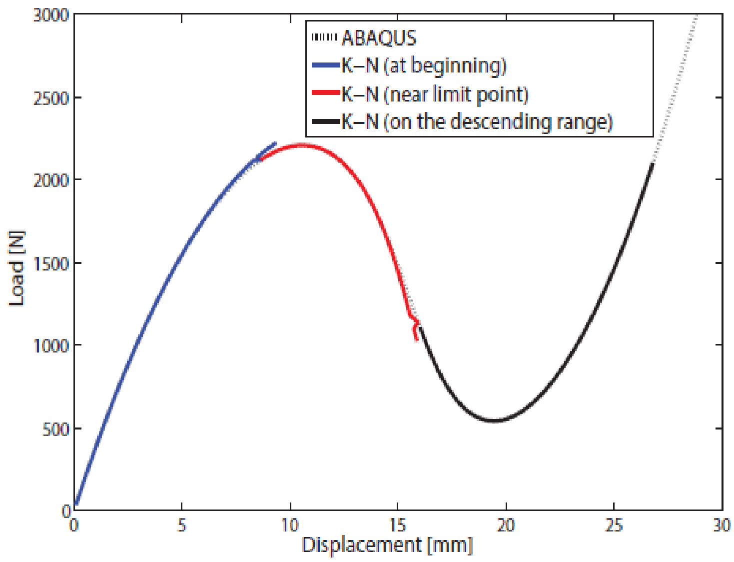


*Fig. 2 Response curve of the Scordelis-Lo shell*

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## REFERENCES

[1] K. Liang, A Koiter-Newton arc-length method for buckling sensitive structures. PhD thesis, Delft University of Technology. July 2013. ISBN 978-94-6203-387-0.

[2] K. Liang, M. Abdalla, Z. Gurdal (2013). A Koiter-Newton approach for nonlinear structural analysis. International Journal for Numerical Methods in Engineering, 96(12): 763-786.