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THICKNESS-SHEAR VIBRATION ANALYSIS OF RECTANGULAR QUARTZ PLATES BY AN IMPROVED NUMERICAL EXTENDED KANTOROVICH METHOD

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Piezoelectric crystals are widely used to make acoustic wave resonators as a frequency standard for time-keeping, frequency generation and operation, telecommunication, and sensing [1]. A large portion of quartz crystal resonators, usually in circular or rectangular platform with all edges free, are operated with the so-called thickness-shear modes of a plate [2]. Considerable efforts have been spent in analysis of piezoelectric resonator with approximate and numerical methods since Mindlin [3, 4]. Although accurate and reliable results have been obtained, with emerging problems and practical needs, precise analysis of thickness-shear vibrations of crystal plates is even more important now [2].

Thickness-shear vibration is characterized by high frequency vibration that needs huge computational cost. There are considerable mathematical difficulties in modelling the problem through three-dimensional (3D) numerical or analytical techniques. Therefore, most literatures in the past decades were based on the 2D equations for the vibrations of elastic plates established by Mindlin [3]. Even with the 2D equations, the parallel computing techniques on Linux clusters are necessary for numerical analyses [5], and many simplifications or approximations are necessary for analytical analyses [6].

Liu et al. [4] proposed a highly efficient numerical method called numerical extended Kantorovich method (NEKM) that reduced the two-dimensional elasticity problem into two one-dimensional ones and then solved them iteratively as the Kantorovich method. But all the processes of reduction and solvation in NEKM were carried out by using a differential quadrature finite element method (DQFEM) [7]. The NEKM could provide the “best” beam approximations, but it still could not satisfy the boundary conditions very well.

Because the DQFEM is highly accurate, the accuracy of the NEKM could be further improved. According to the extended Kantorovich method [8], the deflection w, and the angles of rotations ϕ and ψ could be written into two uncoupled functions as

$$w=\sum_{i=1}^{n} w_{xi}(x)\,w_{yi}(y),\quad \phi=\sum_{i=1}^{n}\phi_{xi}(x)\,\phi_{yi}(y),\quad \psi=\sum_{i=1}^{n}\psi_{xi}(x)\,\psi_{yi}(y) \tag{1}$$

Liu at al. [4] only considered $n = 1$. This paper take n as arbitrary integer to improve the accuracy of the NEKM. The improved NEKM satisfy the completely free boundary conditions much better than [4] as shown in the mode for M_{xy} in figure 1.

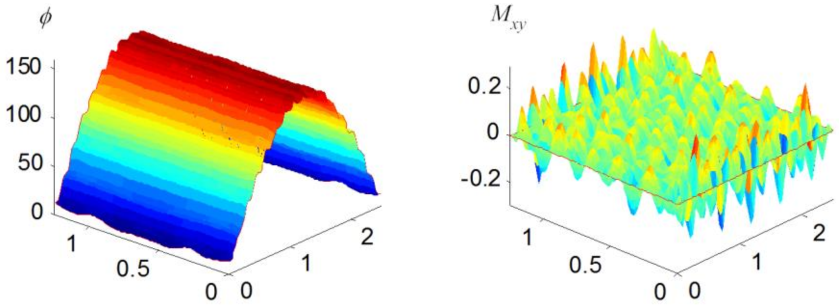

Figure 1. Mode shapes for angles of rotation ϕ and moment M_{xy} of the fundamental thickness-shear mode.

The improved NEKM is a numerical method that is much simpler than the Kantorovich method [6] that does all the processes analytically with great complexity. The high accuracy of the NEKM were validated through numerical comparison with the results in literatures and obtained through other methods. The application of the improved NEKM to thickness-shear vibration of a rectangular crystal plate showed that the method was a promising method for solid mechanics problems that had huge computational cost and required high accuracy.

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