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A DYNAMIC PROGRAMING ALGORITHM FOR CONSTRUCTION OF A CLASS OF OPTIMAL ELIMINATION TREES FOR MULTI-FRONTAL SOLVER ALGORITHM EXECUTED OVER H REFINED GRIDS

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In this paper we present our dynamic programming based algorithm finding optimal elimination trees for computational grids obtained from *h* adaptive finite element method [1]. The elimination tree is a core part of the multi-frontal direct solver algorithm [2], defining the order of elimination of nodes as well as the pattern for construction and merging of the frontal matrices, if the input for the solver algorithm are partially assembled element frontal matrices, not the fully assembled global problem. In other words the input for the multi-frontal solver algorithm is an elimination tree and element frontal matrices. Having the elimination tree we can estimate exactly the computational cost of the multi-frontal solver algorithm. Based on that principle we construct a dynamic programming algorithm constructing a class of elimination trees for a given mesh, and we select the optimal tree, namely the one that has minimum computational cost estimate. This optimization algorithm can be utilized as the learning tool for construction of a heuristic algorithms for a class of refined grids.
The elimination trees found by our optimization algorithm have been compared with the elimination trees obtained from nested dissection algorithm [3], and for some cases, including the edge singularity in two or three dimensions, we have found that our elimination tree provides better computational cost. This is because the optimal elimination tree for such the cases can be obtained only by considering not equally weighted partitions of the graph representing the mesh connectivities.

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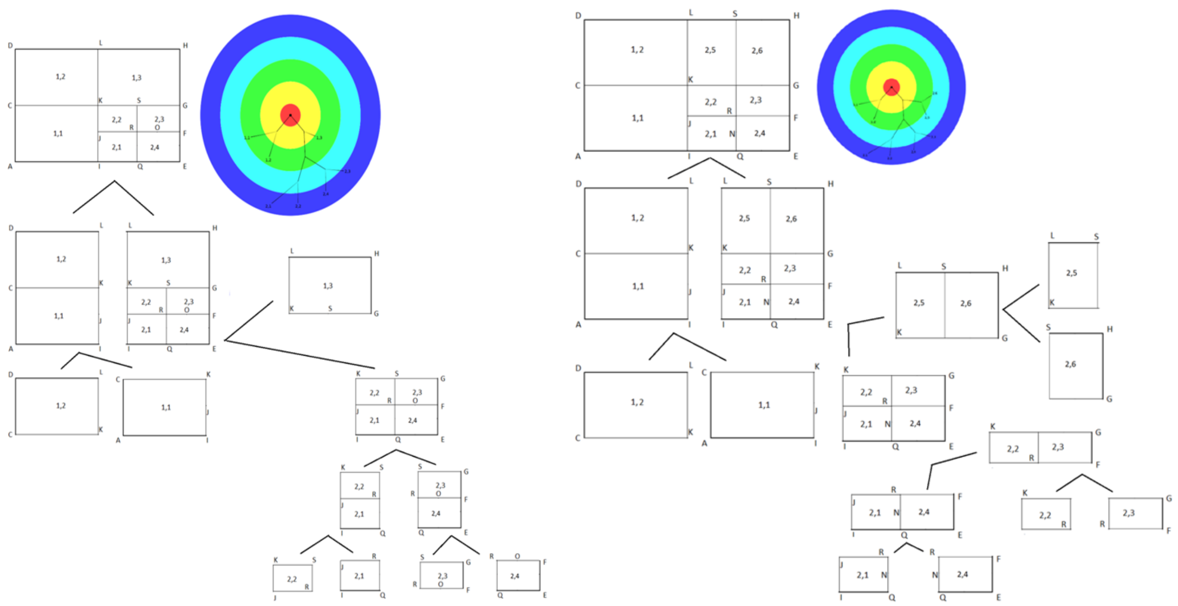

Figure 1. Left panel: The optimal tree found for the mesh with point singularity. **Right panel:** The optimal tree for the mesh with point and anisotropic edge singularity.

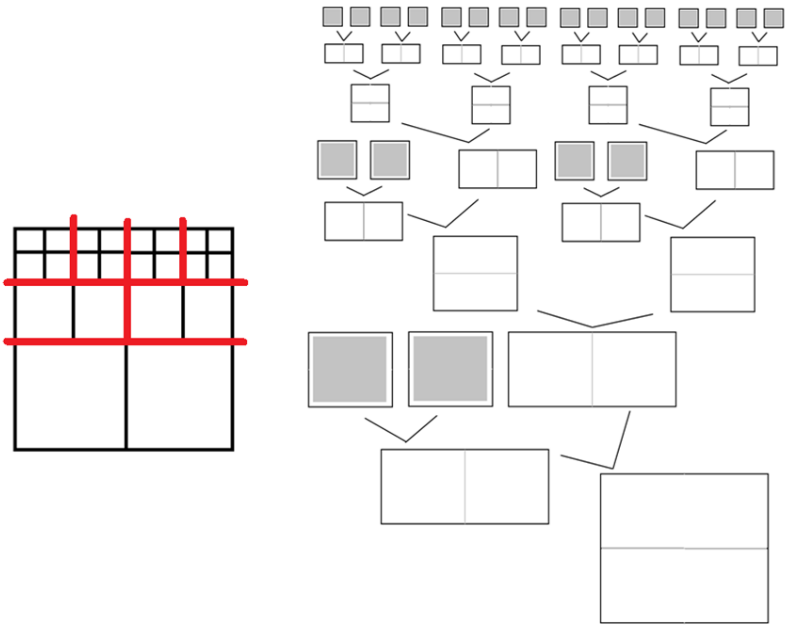

Figure 2. The optimal tree found for the mesh with isotropic edge singularity.

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