

## CONSISTENCY-BASED COUPLING OF ISOGEOMETRIC AND MESHFREE APPROXIMATIONS

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In order to take advantage of the geometry exactness of isogeometric analysis [1] and the refinement flexibility of meshfree method [2], a consistency-based coupling of isogeometric analysis and meshfree approximations is presented. It is shown that unlike the reproducing kernel meshfree shape functions which satisfy the reproducing conditions with the nodal points as the reproducing locations, the monomial reproducing points for different orders of B-spline basis functions in isogeometric analysis are different. We define the reproducing or consistency condition for B-spline basis functions as follows [3]:

$$\sum_{a=1}^n N_a^p(\xi)(\xi_a^{[l]})^l = \xi^l \quad (1)$$

where  $\xi$  is the parametric coordinate,  $n$  is the number of B-spline basis functions,  $N_a^p$  is the  $p$ -th order B-spline basis function,  $\xi_a^{[l]}$  is the reproducing point for the monomial  $\xi^l$ . In meshfree approximation  $\xi_a^{[l]}$  is the same as the meshfree particle  $\xi_a$ , i.e.,  $\xi_a^{[l]} = \xi_a$ . For B-spline basis functions, the reproducing point  $\xi_a^{[l]}$  is defined as [3]:

$$\xi_a^{[l]} = \sqrt[p]{S_p^l[G_{a+1}^{a+p}] / C_p^l}, \quad C_p^l = p! / [l!(p-l)!] \quad (2)$$

where  $G_{a+1}^{a+p} = \{\xi_{a+1}, \xi_{a+2}, \dots, \xi_{a+p}\}$ .  $S_p^l[G]$  that means selecting  $l$  elements from  $G$  and multiplying them together as one term, and then summing all the possible terms together. For example, the first and second order reproducing points for quadratic B-spline basis functions can be obtained from Eq. (2) as:

$$\begin{cases} S_2^1[G_{a+1}^{a+2}] = \xi_{a+1} + \xi_{a+2}, & S_2^2[G_{a+1}^{a+2}] = \xi_{a+1}\xi_{a+2} \\ \xi_a^{[1]} = \sqrt[2]{S_2^1[G_{a+1}^{a+2}] / C_2^1} = (\xi_{a+1} + \xi_{a+2}) / 2, & \xi_a^{[2]} = \sqrt[3]{S_2^2[G_{a+1}^{a+2}] / C_2^2} = \sqrt{\xi_{a+1}\xi_{a+2}} \end{cases} \quad (3)$$

The coupling of isogeometric and meshfree approximations is established according to the following consistency requirement [3-5]:

$$\sum_{A=1}^N \Phi_A(\xi) p(\xi_A^{[\cdot]}) = \sum_{A=1}^{NC} R_A(\xi) p(\xi_A^{[\cdot]}) + \sum_{B=1}^{NP} \Psi_B(\xi) p(\xi_B^{[\cdot]}) = p(\xi) \quad (4)$$

where  $R_A$  is the NURBS basis function which is identical to the B-spline basis function when uniform weight is used,  $NC$  is the number of NURBS or B-spline basis functions and  $N$  is the total number of shape functions for the coupled approximation.  $p(\xi_A^{[\cdot]})$  is a mixed

reproducing point vector consisting of  $\xi_a^{[l]}$ 's to ensure arbitrary order monomial reproducibility for both B-spline basis functions and meshfree shape functions:

$$p(\xi_A^{[\cdot]}) = \{1, \xi_A^{[1]}, \eta_A^{[1]}, (\xi_A^{[2]})^2, \xi_A^{[1]}\eta_A^{[1]}, (\eta_A^{[2]})^2, \dots, (\xi_A^{[l]})^l, \dots, (\eta_A^{[m]})^m, \dots, (\xi_A^{[p]})^p, \dots, (\eta_A^{[p]})^p\}^T \quad (5)$$

Assuming a reproducing kernel form for the meshfree shape function  $\Psi_A$  with the combination of Eq. (4) yields:

$$\Psi_A(\xi) = p^T(\xi_A^{[\cdot]}) \bar{M}^{-1}(\xi) [p(\xi) - g(\xi)] \phi(\xi_A^{[1]} - \xi), \quad g(\xi) = \sum_{A=1}^{NC} R_A(\xi) p(\xi_A^{[\cdot]}) \quad (6)$$

where  $\phi$  is the cubic B-spline kernel function. The coupling of isogeometric and meshfree approximations is systematically illustrated in Fig. 1. Thereafter a coupled isogeometric-meshfree Galerkin method is formulated which is shown to be capable of achieving the expected convergence rates as shown in Fig. 2.

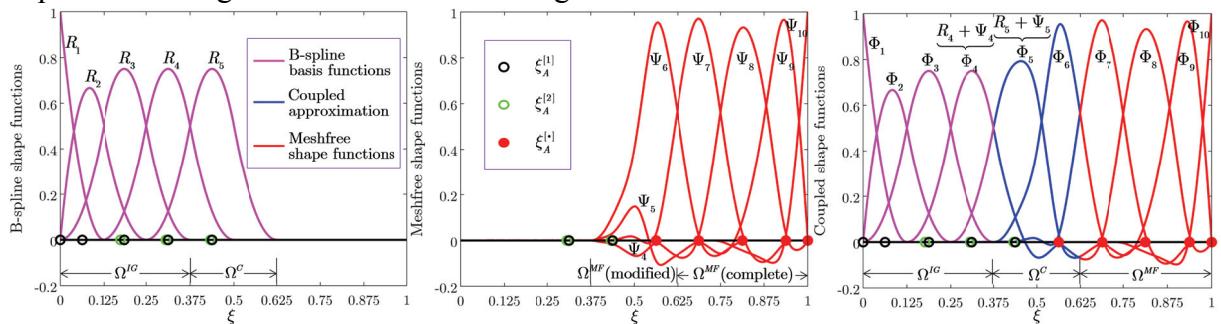


Figure 1 Illustration of the coupling of isogeometric and meshfree approximations

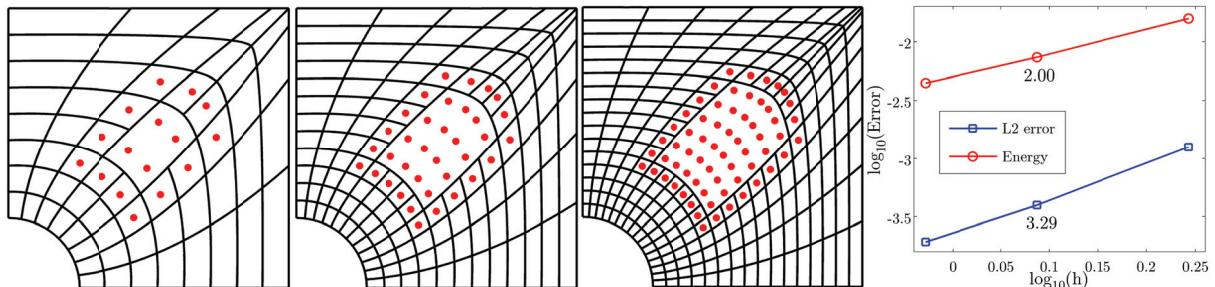


Figure 2 Convergence of the quadratic coupled method for the infinite plate with a circular hole problem

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