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INFO-GAP ANALYSIS FOR NUMERICAL UNCERTAINTY ASSOCIATED WITH TRUNCATION ERROR

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Quantification of the uncertainty associated with the truncation error implicit in the numerical solution of discretized differential equations remains an outstanding and important problem in computational science. Analysis based on Richardson extrapolation is the usual approach to characterizing the uncertainty associated with discretization error effects for such numerical solutions. Ideally, computed solutions on sufficiently many different discretized grids are available to completely evaluate an extrapolated solution. Implicit in such analyses is the assumption that the computed solutions are in the domain of asymptotic convergence, i.e., that the mesh discretization is sufficiently fine for the assumption of power law dependence of the discretization error on the mesh length scale to be valid. Such an application of Richardson extrapolation requires that all solutions be in this domain.

We instead focus on the extremely uncertain case where only two calculations are available. Consequently, there is no direct evidence that these solutions are in the domain of asymptotic convergence. This situation corresponds to the practical reality commonly faced in many engineering-scale simulations. Because of the limited amount of information available in this information-poor case, the usual Richardson extrapolation analysis cannot be rigorously justified, much less directly utilized.

We propose an application of info-gap theory to this problem. Info-gap theory [1] provides a method for supporting model-based decisions under severe uncertainty. Here, an "info-gap" is a disparity between what is known and what needs to be known in order to inform a decision. In the case of numerical uncertainty associated with discretization error, "what is known" is the numerical solution computed at different mesh resolutions together with an algorithm by which these values are extrapolated to the nominally infinitely refined mesh, while "what needs to be known" is the correct extrapolation algorithm (either the form of this function or the parameters in the assumed form) that would produce the numerical solution that corresponds to an infinitely well-resolved calculation. We apply info-gap concepts to this highly underdetermined problem and demonstrate the kind of insight this analysis can provide.

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