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# STATISTICAL MODELING OF DAMAGE IN MATERIALS WITH RANDOMLY DISTRIBUTED ANISOTROPIC INCLUSIONS

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In the present contribution, a statistical model [1] describing the coupled deformation and damage of materials with randomly distributed anisotropic inclusions is proposed. The mechanism of micro-damage of such composites is studied assuming that the micro-strength of the material is inhomogeneous. A single micro-damage is modeled by an empty quasi spherical pore forming in place of a damaged micro-volume. A composite material of stochastic structure with porous components is obtained, whose damage is described as increasing porosity. The macro-stresses and macro-strains of such a composite will be related by Hooke's law:

$$\overline{\boldsymbol{\sigma}} = \mathbf{C}^* : \overline{\varepsilon} \,, \tag{1}$$

where $\mathbf{C}^*$ is the tensor of effective elastic constants, which can be determined by the method of conditional moments (MCM) [2, 3]. An overbar denotes statistical averaging. Assuming that the damage occurs in the matrix reinforced by unidirectional ellipsoidal inclusions, the overall elastic moduli of such a composite are functions depending on the elastic moduli of the components $\mathbf{C}_1$, $\mathbf{C}_2$ (1 and 2 refer to the inclusions and to the matrix, respectively), the matrix porosity $p_2$, the inclusion volume fraction $c_1$, and the aspect ratio of the ellipsoids $s$ [2]

$$\mathbf{C}^* = \mathbf{C}^*(\mathbf{C}_1, \mathbf{C}_2, c_1, p_2, s), \quad s = s_1/s_3 \,. \tag{2}$$

The formation of micro-damage under loading is described by a damage criterion for the micro-volume, which is given as a limiting value of the intensity of average shear stresses occurring in the undamaged part of the matrix material [1]

$$I_\sigma^2 = \left( \overline{\boldsymbol{\sigma}}^{2\prime}, \overline{\boldsymbol{\sigma}}^{2\prime} \right) = k_2 \,. \tag{3}$$

The average stresses occurring in the undamaged part of the matrix material can be determined as a function of the macro-deformations of the entire composite (see details in [1])

$$\overline{\boldsymbol{\sigma}}^2 = \left( \widetilde{\mathbf{C}}^* : \overline{\varepsilon} \right) \Big/ \left(1 - p_2\right), \quad \widetilde{\mathbf{C}}^* = \mathbf{C}_2 \left[ \mathbf{I} + (1-c_1)\left(c_1\mathbf{C}_1 + (1-c_1)\mathbf{C}_2 - \mathbf{C}^*\right)(\mathbf{C}_1 - \mathbf{C}_2)^{-1} \right] . \tag{4}$$

The corresponding limit value of an equivalent stress from the damage criterion Eq.(3) is considered as a random function (statistically homogeneous) of coordinates. The one-point distribution function of this limit value is given by a Weibull distribution [1]

$$F(k_2) = \begin{cases} 0 \,, & k_2 < k_{02} \\ 1 - \exp\left(-m_2 (k_2 - k_{02})^{\alpha_2}\right), & k_2 \ge k_{02} \end{cases} \,, \tag{5}$$

where $k_{02}$ is the lower limit value of the intensity of the tangential stresses $k_2$ at which the damage occurs within some macro-volume of the matrix, and $k_2$, $m_2$ and $\alpha_2$ are the parameters chosen from a condition of the best approximation of the strength distribution.

The general property of the one-point distribution function of the ergodic random field of ultimate micro-strength is used to derive the damage evolution equation [1]:

$$p_2 = p_2^0 + F\left(I_\sigma^2\right)\left(1 - p_2^0\right). \tag{6}$$

here $p_2^0$ is initial porosity of the matrix. Stresses in the undamaged part of the matrix material $\bar{\boldsymbol{\sigma}}^2$ can be expressed as a function of macro-strains of the entire composite $\bar{\varepsilon}$, then these equations enable to determine the current porosity of the matrix $p_2$, generated by micro-damage, from equations (1) - (6), as a function of macro-stresses $\bar{\boldsymbol{\sigma}}$ or macro-strains $\bar{\varepsilon}$.

The Eqs. (2) - (6) for determining the effective elastic moduli of a porous transversally-isotropic composite and the damage evolution equation are used as the fundamental relations. It allows us to describe the deformation, the damage and their mutual influence on the deformation properties of such a composite. The MCM [2, 3], the damage evolution equations [1], and the Newton-Raphson method are used to set up an algorithm to calculate the effective deformation characteristics of composite materials depending on macro-strains. The effect of the material damage on the macro-stress – macro-strain relationship of the entire composite is established. On the basis of a numerical solution, dependences of matrix porosity (damage) and macro-stresses on strains of the material representing the epoxy reinforced with unidirectional ellipsoidal carbon fibers for the case of uniaxial tension ($\bar{\varepsilon}_{11} \neq 0$) are constructed.

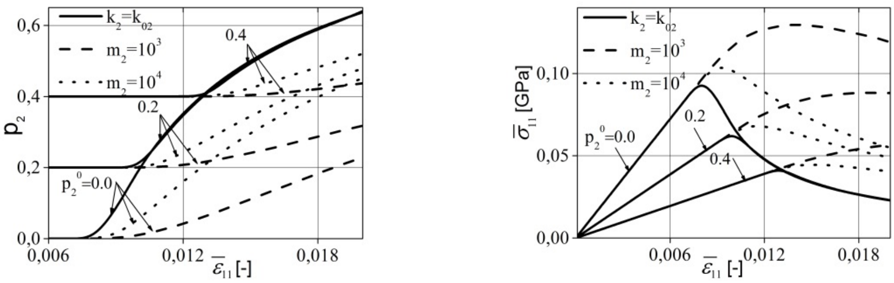


Fig. 1: Damage (left - $p_2$) ; stresses (right - $\bar{\sigma}_{11}$) as functions of unidirectional loading strains $\bar{\varepsilon}_{11}$ (predicted).

It can be seen that the macro-stress–macro-strain curves are strongly dependent on the parameters of the strength distribution function $F(k_2)$. As the parameter $m_2$ increases, the macro-stress at a given macro-strain decreases and the behavior of the curves tends to that of the curves that disregard the strength scatter in micro-volumes of the matrix material (solid line).

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