

## Development of a transient structural analysis algorithm by using FETI-local method

JunYoung Kwak<sup>1</sup>, Haeseong Cho<sup>1</sup>, SangJoon Shin<sup>1</sup> and Olivier A. Bauchau<sup>2</sup>

<sup>1</sup> School of Mechanical and Aerospace Eng Seoul National Univ, San 56-1, Shillim-Dong, Gwanak-gu, Seoul, [kjy84@snu.ac.kr](mailto:kjy84@snu.ac.kr), [nicejjo@snu.ac.kr](mailto:nicejjo@snu.ac.kr), [ssjoon@snu.ac.kr](mailto:ssjoon@snu.ac.kr)

<sup>2</sup> Dept. of Mechanical Eng UM-SJTU, 800 Dong Chuan Road Shanghai 200240 China, [Olivier.bauchau@sjtu.edu.cn](mailto:Olivier.bauchau@sjtu.edu.cn)

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### 1. Introduction

Advancement of the computer hardware and software technologies allows it possible to attempt large-size analysis in the field of the multi physics. Consequently, development of effective solution methodologies for large-size structural problems is of increasing importance. The finite element tearing and interconnecting (FETI) method is probably the most commonly used domain decomposition approach for large-scale structural analysis. It was proposed by Farhat and Roux [1] as a parallel solution algorithm for elliptic partial differential equations. The basic idea of FETI was to decompose the computational domain into non-overlapping sub-domains. Lagrange multipliers were used to enforce compatibility of the degrees of freedom along the interfaces between the sub-domains. The approach was extended to fourth-order elasticity problems (two-level FETI) partial differential equations. More recently, the dual-primal FETI (FETI-DP) [2] method was proposed. In the present paper, the augmented Lagrangian formulation will be built into its solution process, and thus will not need to evaluate pseudo-inverse of the floating sub-domain stiffness matrices. Because all the sub-domain stiffness matrices are non-singular, standard column solvers will be used directly. A time transient computational algorithm will be developed by extending the existing FETI method [3]. The computational performance of the proposed transient formulation will be compared with those by the Dual-primal FETI algorithm.

### 2. Description of the proposed domain decomposition algorithm

In the dynamic analysis based on the proposed FETI, the equation of motion at time  $t$  is as follows.

$$[M]\{\ddot{u}\}^t + [C]\{\dot{u}\}^t + [K]\{u\}^t = \{F\}^t \quad (1)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the system mass, damping, and stiffness matrices, respectively. In this paper, the system was assumed to be without damping. The solution procedure of proposed dynamic FETI algorithm can be summarized as follows.

1. Compute the system matrices, such as  $[M]$  and  $[K]$ .
2. Solve for an initial acceleration.

$$\ddot{u}_0 = [M]^{-1}(F_0 - Ku_0) \quad (2)$$

where  $u_0$  is the initial displacement vectors.

3. Repeat Eqs. (3) through (5) for each time step. To solve Eq. (3), we use the proposed FETI method [3].

$$F^{\text{eff}}_{u_{n+1}} = F_{n+1} + M\left[\frac{4}{\Delta t^2}u_n + \frac{4}{\Delta t}\dot{u}_n + \ddot{u}_n\right] \quad (3)$$

where  $F^{\text{eff}} = \frac{4}{\Delta t^2} M + K$ .

$$\ddot{u}_{n+1} = \frac{4}{\Delta t^2} (u_{n+1} - u_n - \Delta t \dot{u}_n) - \ddot{u}_n \quad (4)$$

$$\dot{u}_{n+1} = \frac{2}{\Delta t^2} (u_{n+1} - u_n) - \dot{u}_n \quad (5)$$

### 3. Numerical results

The proposed dynamic FETI method is applied to the solution of a dynamic two-dimensional plane stress problem and its predictions are compared with those obtained by the dynamic FETI-DP. In Fig. 1, the tip deflection shows oscillatory amplitude with respect to the static deflection. And it shows a comparison results between the proposed dynamic FETI and FETI-DP. For the proposed approach, the time transient analysis show good agreement with those obtained by FETI-DP. The proposed transient FETI-local method will be extended to the solution of three-dimensional dynamics problems.

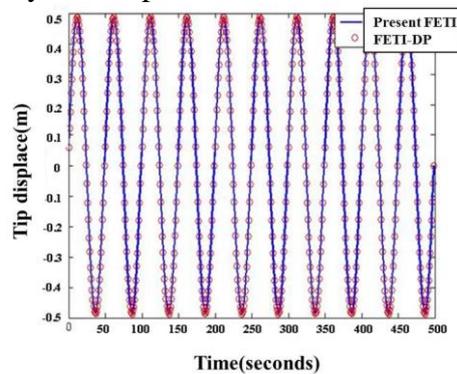


Fig. 1. Time response using Newmark method (500 step)

### 4. Conclusion

This paper describes development of a time transient computational algorithm based on a finite element domain decomposition technique for the analysis of large scale structural dynamics problems. For that, the proposed FETI method is described and the resulting dynamic analysis combined Newmark integration scheme and the proposed FETI algorithm. During the 500 time steps, both analyses show good agreement within a difference of 0.01%.

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