

AN UPDATED LAGRANGIAN METHOD WITH ERROR ESTIMATION AND ADAPTIVE REMESHING FOR VERY LARGE DEFORMATION ELASTICITY PROBLEMS

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Accurate simulations of large deformation hyperelastic materials by the finite element method is still a challenging problem. In a total Lagrangian formulation, even when using a very fine initial mesh, the simulation can break down due to severe mesh distortion. Error estimation and adaptive remeshing on the initial geometry are helpful and can provide more accurate solutions (with a smaller number of degrees of freedom) but are not sufficient to attain very large deformations. The updated Lagrangian formulation where the geometry is periodically updated is then preferred. Remeshing may still be necessary to control the quality of the elements and to avoid too severe mesh distortion. It then requires frequent data transfer from the old mesh to the new one and this is a very delicate issue. If these transfers are not done appropriately, accuracy can be severely affected.

In this paper, we present an updated Lagrangian formulation where the error on the finite element solution is estimated and adaptive remeshing is performed in order to concentrate the elements of the mesh where the error is large, to coarsen the mesh where the error is small and at the same time to control mesh distortion. In this way, we can reach high level of deformations while preserving the accuracy of the solution. Anisotropic remeshing pushes this idea one step further by allowing the presence of elements with large aspect ratio in certain directions compatible with the solution. This also reduces the number of DoF needed to obtain a given accuracy. The mesh is thus adapted in order to both improve the accuracy of the numerical solution and avoid degenerate elements, while also decreasing the computational burden. In this work, the fully optimal anisotropic mesh

adaptation method based on a hierarchical error estimator developed by Bois et al. [1, 2] is used to estimate the error and adapt the mesh accordingly.

Once a remeshing step has been completed, the deformation gradient tensor, which keeps track of the previous deformations in an updated Lagrangian method, needs to be transferred from the old mesh to the new mesh. Special attention is thus also given to data transfer methods and a very accurate cubic Lagrange projection method is introduced. This projection method has not only shown to be very accurate but also very robust in the problems that we have considered.

As large deformation problems frequently have highly nonlinear solutions, the Moore-Penrose continuation method is used to automatically pilot the complete algorithm including load increase, error estimation, adaptive remeshing and data transfer. A new approach for the implementation of the Moore-Penrose continuation method, facilitating the detection of bifurcation points, will also be presented.

A number of examples, similar to the one illustrated in Figure 1, will be presented and analyzed.

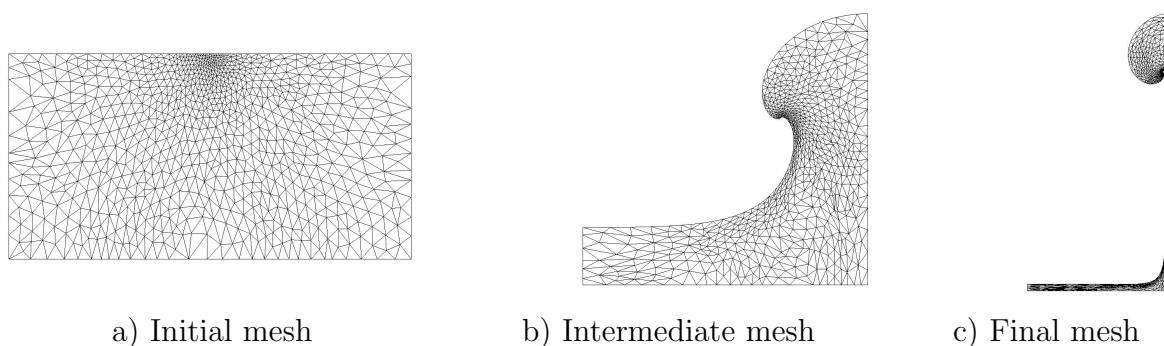


Figure 1: Example of very large deformation problem: deformed meshes at different instances during the simulation (note: scale is different for each subfigure).

REFERENCES

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