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LINEAR COMPUTATIONAL COST GALOIS BASED GRAPH GRAMMAR DIRECT SOLVER FOR H ADAPTIVE GRIDS

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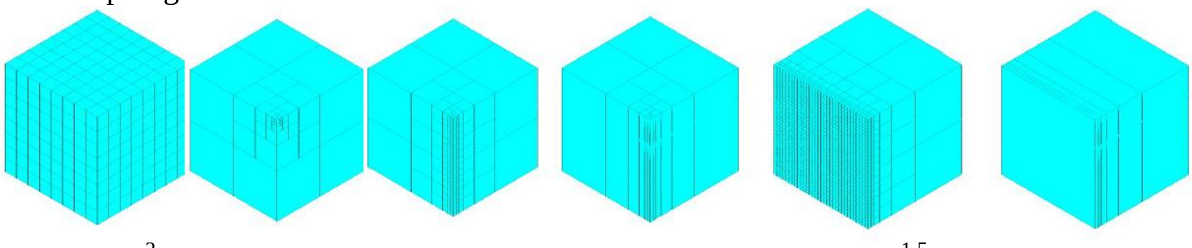
In this paper we construct a linear computational cost multi-frontal direct solver for two and three dimensional *h* adaptive grids.
In two dimensional h adaptive computations for rectangular elements, there are three basic ways of element refinement: the element can be uniformly refined, it can be refined towards point or edge, in isotropic way, or towards edge in anisotropic way.
In three dimensional h adaptive computations for cubic elements, there are six basic ways of element refinement: the element can be uniformly refined, it can be refined towards point or edge or face, in isotropic way, towards edge or face in anisotropic way. This is illustrated in Figure 1. The other refinements are composed by the mixture of these three or six basic ways, respectively. We show theoretically, that most of the mentioned above 2D and 3D cases, can be solved by multi-frontal direct solver algorithm in a linear cost, if we construct the elimination tree in a proper way. In other words, in this paper we advocate the usage of adaptive grids, since they reduce significantly the cost of the direct solver algorithm. The linear cost of the solver algorithm doesn't depend on the PDE being solved, it only depends on the topological structure of the mesh.



$O(N^2)$ O(N) O(N) O(N) $O(N^{1.5})$ O(N)

Figure 2. The six basic ways of refinement of a three dimensional cubic element: uniform refinement, isotropic point refinement, isotropic edge refinement, anisotropic edge refinement, isotropic face refinement, anisotropic face refinement. The computational costs of direct solver algorithm executed over the grids.

The multi-frontal solver algorithm can be expressed by a sequence of graph grammar productions [1, 2]. Namely, the solver algorithm works over the elimination tree, and the construction and processing of nodes of the elimination tree can be expressed by graph grammar productions. Expressing the multi-frontal solver algorithm by a sequence of graph grammar productions allows for identification of sets of graph grammar productions that can be executed in parallel. The resulting solver algorithm works over the binary elimination tree in parallel, processing the tree level by level from the leaves up to the root, and back from the root down to the leaves. We present graph grammar productions for construction of the elimination trees for h refined two or three dimensional computational grids, and for execution of the multi-frontal solver algorithm over the trees. The graph grammar productions describing the solver algorithm have been implemented and executed using the Galois [3] system. The Galois is a system that automatically executes graph grammar productions *in parallel* on shared-memory machines.

The resulting graph grammar based solver algorithm has been compared to sequential MUMPS solver, for several two and three dimensional cases. The sequential version of our solver has similar linear performance like MUMPS solver, however the shared memory parallel version scales very well and outperforms MUMPS solver, compare Figure 1.

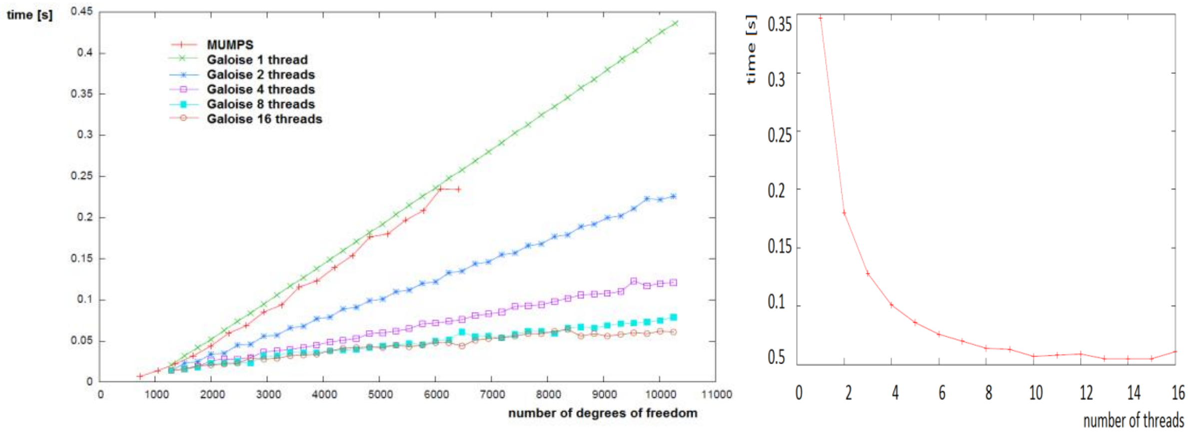


Figure 2. Left panel: Comparison of Galois based graph grammar solver with MUMPS solver, for 1,2,4,8 and 16 threads, for a single three dimensional element with isotropic refinements towards point. **Right panel:** Scalability of 16 threads Galois based graph grammar solver, for three dimensional mesh with 8 elements with central point singularity.

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