Fractal Methods in Coastal Diffusion Models

J.M. Redondo[(1)], M. Diez[(2)] and P. Fraunie[(3)]

[(1)] Univ. Politecnica de Catalunya, Dept. Fisica Aplicada, Barcelona, Spain
[(2)] Ports de la Generalitat, , 08800, Vilanova i la Geltru, Barcelona, Spain
[(3)] Univ. Sud Toulon Var. M.I.O. La Garde, France

Abstract

The satellite-borne SAR and ASAR seems to be an excellent system not only to detect man-made oil spills and tensioactive slicks but it also detects dynamic features and the ocean eddies of different sizes. The study of the topology of the regions of different rugosity of the ocean can map the vortical, eliptical regions as well as the hyperbolic shear dominated areas, is also a convenient tool to investigate the eddy structures, the scale to scale energy and enstrophy transfer of a certain area, and to calculate the eddy diffusivity values. The effect of bathymetry and local currents are important in describing the ocean surface behavior. In the NW Mediterranean the maximum eddy size agrees remarkably well with the limit imposed by the local Rossby deformation radius using the usual thermocline induced stratification, Redondo and Platonov (2001). The Rossby deformation radius, is attained when buoyancy and Coriolis forces are in equilibrium, and it is defined as Rd = (N/f)h, where N is the Brunt-Vaisalla frequency, f is the local Coriolis parameter (f=2 Ω *sen* φ, *wh*ere Ω is the rotation of the earth and a is the latitude) and h is the thermocline depth, Rd is about 6-20 Km.

The geometry of gray scale ranges and boundaries of spatial dynamic surface features may contain new helpful information. Already we used multi-fractal analysis techniques to investigate man-made oil spills, Redondo and Platonov(2009), Platonov et al.(2008). We now work in the aplication of these techniques to the analysis of ocean surface multi-fractal features (eddies, mushroom-like currents, etc.) to understand the scale to scale transport. (Benjamin et al 1998, Redondo et al. 2008, Diez et al. 2008).

1-Introduction

The model described to model the 2D surface of the ocean near the coast is based on stochastics methods, which have the objection that don't take into account the real ocean flow profile. On the other hand, there are many ways to simulate a fluid flow, but when this is turbulent, these simulations become complicated, expensive and inaccurate. Our aim is to simulate the behaviour of marked tracer particles in a turbulent flow, in a simple way with a kinematically simulated model and to validate the results. We use the Kinematic Simulation (KS) model, suggested by Kraichnan (1966,1970) and developed by Fung et al.(1990).

In this model, velocity field is generated through a Fourier series of random modes. The typical scales and the energy spectrum of the turbulence are inputs of the model. As we do not solve the flow in a discrete grid, but use a random predictive expression, we can simulate the flow at the smallest scales.

2-Stratified parameters

The dynamical processes associated with the stably stratified flow such as what occurs in the ocean thermocline are less well understood than those of its convective counterparts. This is due to its complexity, and the fact that buoyancy reduces entrainment across density interfaces. We present results on a numerical simulation of homogeneous and density stratified fluids and of comparable laboratory experiments where a sharp density interface generated by either salt concentration or heat, advances due to grid stirred turbulence. We parametrize the level of buoyancy at the density interface by a local Richardson number defined as

$$Ri = \frac{g\Delta\rho l}{\rho u'^2} \qquad (1)$$

where g is gravity, dr is the density difference across the interface, which may be due to a temperature or salinity jump. l is the integral lengthscale and u' is the r.m.s. velocity scale.

It is also important to investigate the relationship between the Flux Richardson number and the Gradient or local one and the ways in which the interface extracts energy from the turbulence source via internal waves. Internal gravity (or buoyancy) waves are characteristic of the stable boundary layer and contribute to its transport processes, both directly, and indirectly via internal wave-induced turbulence. These processes are able to control entrainment across strong

density interfaces. A comparison of the range of entrainment values from laboratory experiments with those ocurring in nature, both in the atmosphere and ocean shows the importance of modeling correctly the integral lengthscales of the environmental turbulence.

As the Entrainment may actually be related to the ratio of the flux to gradient Richardson numbers as well as the Turbulent Schmidt or Prandtl number, $Sc = Km / Kh$.

$$Rf = E\,Ri = Kh\,Ri / Km \qquad (2)$$

So vertical mixing or diffusion is much smaller than horizontal one. There is a very strong reduction in vertical scalar diffusivity as a function of stratification if it is stable, the opposit is true if the scalar field is gravitationaly unstable.

3- KS numerical model

A series of Kinematic simulations have studied the diffusion in homogeneous, non stratified flows as well as the dispersion of dense particles within a turbulent stratified flow. detailled also in Castilla (2001) .

The velocity calculation in any point $\vec{x}$ in a determined instant t is made with the expression

$$\vec{\mathrm{v}} = \sum_{n=0}^{N} \left[\vec{A}_n \sin(\vec{\kappa}_n \vec{\mathrm{x}} + \omega_n t) + \vec{B}_n \cos(\vec{\kappa}_n \vec{\mathrm{x}} + \omega_n t) \right] \qquad (3)$$

where v_n is the turnover velocity of the eddy with scale l_n, and is related to the energy through

$$\omega_n = \lambda\sqrt{\kappa_{ij}^3 E(\kappa_n)} \qquad (4)$$

which is a parameter which indicate the turbulence stationarity degree. The vectors $\vec{\kappa}_n$ have random direction and sense, and its modula depends on the discretization on phase space. The vectors $\vec{A}_n$ and $\vec{B}_n$ have also random direction and sense, but they must be normal to $\vec{\kappa}_n$.

The simulation is then controlled by the integral and Kolmogorov scales, the molecular viscosity of fluid v, and the spectrum shape within the inertial subrange. From the numerical point of view, we divide the wavenumber inertial range in N intervals, which can be calculateds following a geometric progression in order to put, more or less, a comparable energy at each scale. When we have calculated the velocity field, we put a sufficiently high number of particles randomly distributed, with a determined initial standard deviation, and we study its temporal development.

An important characteristic of KS is that we do not need a grid. We can calculate fluid velocity at \emph{any} position and time. To integrate the equations (3,4), we have used a fourth order Runge-Kutta scheme with fixed time step. The time step is a fraction of the characteristic turnover time of the smallest eddies, given by Kolmogorov(1941) and over a range of scales where diffusivity behaves acording to the 4/3 Law, Richardson(1929).

Intuitively, we expect that the size of the cloud grows rapidly when it is of a size imbeded at the inertial range, and then slows down when it reaches and exceeds the integral scale. Let be D the particles set size. If its value is of the order of, or smaller than, the turbulence integral scale , the eddies modify its value in a sizeable way. However, if D is much bigger than the integral scale, this modification doesn't affect much the global growth of the tracer cloud.

Nicolleau, F. y Vassilicos, J. C. (1998) presented the argument that stochastic models of turbulent flow based around the concept of the random walk rely on the assumption that particle paths are Brownian motion paths. However, experimental observations emply that particle paths within turbulent flow are actually smooth, non--Markovian trajectories. This fundamental observational aspect of turbulent flow is mimicked well by Kinematic Simulations (KS), which are Lagrangian models of dispersion that incorporate turbulent-like flow structure in every realisation of the Eulerian velocity field. The principle behind the use of KS is that the dominant property of turbulent flow is the smooth geometry of particle trajectories, and all other aspects of turbulent flow such as departures from Gaussianity that arise from the full solution of the Navier-Stokes equations are secondary.

In an unstratified flow, a KS flow field consists of a random, truncated Fourier representation in space and time, subject to constraints associated with incompressibility, and a prescribed initial energy spectrum. For stratified calculations, two further constraints are imposed, associated with the internal wave field in stratified flows, and the tendency of density variations to suppress vertical motion. With these model modifications, good agreement is found between KS and DNS with regard to the confinement in the vertical direction characteristic of stratified turbulence.

Since stratifed flows exhibit this vertical confinement, KS in strictly two dimensions was considered as a first step to understanding dispersion within a stratified flow. The properties of ensemble averages of the separation between two particles in a 2D turbulent flow were considered, and the KS approach was found to give satisfactory answers, with good comparison to experiment.

A Large Eddy Simulation model (LES) may used to simulate the evolution of the rise of a bubble driven convective structure. This is an interesting problem to investigate the mixing process in the ocean surface layers when an injection of air or CO_2 is used as an imput of mechanical energy. Also taking into account the possible use of the Oceans as a CO2 sink.

Together with the momentum equations, the continuity equation for incompressible flow is used taking into account only x and z components.

4- Results

This work strives to obtain a better understanding and compare several types of direct and remote sensing observations of the effects of surface wind turbulence on diffusivity and local circulation measured in the ocean-atmosphere interface. We obtain statistical information on the size and topological characteristics of eddies ad other features in the gulf of Lions, being able to compare coastal radar based in situ measurements with remote sensing satellite measurements. Here we also address the problem of the effects of background turbulence, eddy diffusivity and vorticity in the coastal areas.
The meteorological phenomena as cyclones, atmospheric fronts, surface wind, atmospheric internal waves and rains are also detected by the SAR images due to their effect on the sea surface roughness, but because the scale of atmospheric processes is quite different to the ocean dynamics, i.e. Rossby deformation radius is more than 10 times larger, as discussed below, different spectral powers, synoptic front behaviour and other effects are distinguished and can be predicted or checked with cloud presence, meteorological visible and infrared images so it is possible to single out most of the typical ocean structures, and among these the spiral vortices (Munk et al. 2000) also detected by sun glitter.
Next we describe the SAR images and the eddy size analysis, in section 3 we present coastal based measurements of vortices in the Rhone river area and in section 4 we show the statistical structure and topology of the detected vortices. Finally we discuss the results and how turbulent diffusivity may be related to spectral eddy information and present the conclusions.

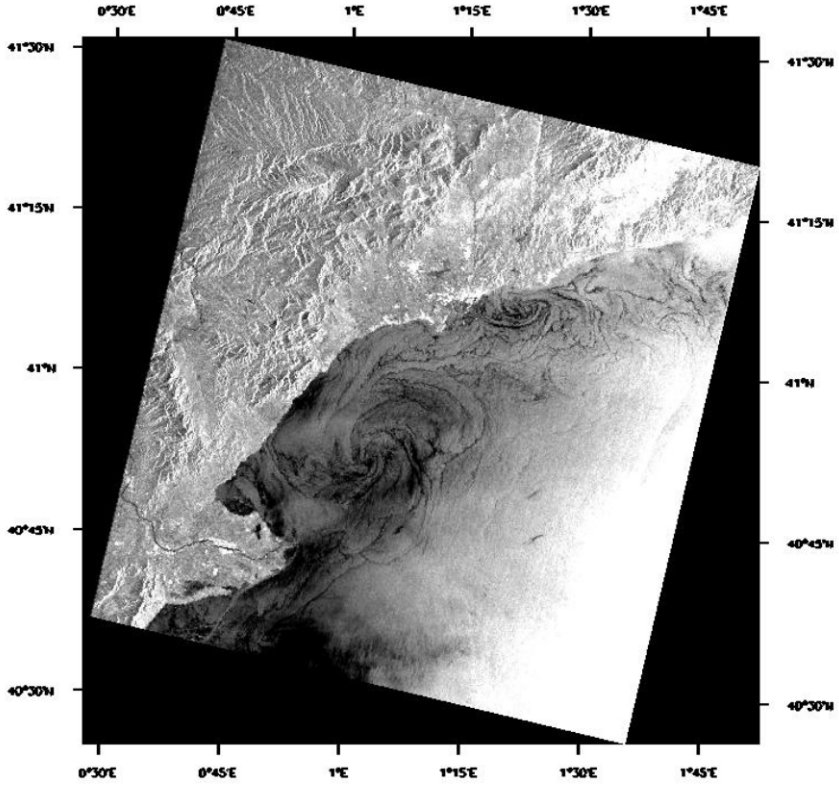

Figure 1: Detection of eddies in the NW Mediterranean by SAR (ERS-1/2) Dynamic features on sea surface near the Ebro delta. ERS-2 SAR 100Km x 100Km image on 27.08.97 at 10:30 UTC.

4.1- Eddy sizes in the ocean surface

In coastal flows the distance to the coast can be assumed as the largest possible eddy length scale L, on the other hand the smallest length scale for a 3D vortex structure, where molecular mixing takes place; L_k can be taken as Kolmogorov (local isotropic turbulence) scale so that $L_k = (\nu^3/\varepsilon)^{1/4}$, being ν the kinematic viscosity and ε the mean rate of turbulent kinetic energy dissipation. A good estimation of ε is $1\text{-}5x10^{-5}$ cm2 $s^{-3}$and for ocean water , the viscosity is $\nu = 10^{-2}$ cm2 s−1, so L*k* lies between 1mm and 1 cm. A similar Kolmogorov timescale, *Tk* is about one hundredth of a second. The longest timescales are not fixed, but considering coastal flows, where the distance to the coast limits the structure of the flow, a few days would be the 2D turbulence synoptic time scale. When the horizontal dimensions of the turbulent motion are much larger than the lengthscales limited by buoyancy (e.g. Thorpe, Ozmidov, Monin-Obukhov, etc.) or. by the depth of the thermocline, then large scale 2D turbulence occurs and the vorticity axis is approximately vertical. So Kolmogorov scaling is no longer valid and a Kraichnan scaling takes place, due to an enstrophy driven cascade.

These features, which are ubiquitous in large scale turbulence, can be generated by circular motion due to several mechanisms: earth rotation, current shear, turbulent wind field, barotropic or baroclinic instabilities, vortexes generated by topography of the ocean bottom, etc... One of the main large scale manifestation of turbulence in the ocean (Munk et al. 2000) are synoptic eddies, which have maximum dimension of around 100 km. in the ocean and about one thousand kilometers in the atmosphere.

In a strictly two-dimensional flow with weak dissipation, energy input at a given scale is transferred to larger scales, because these constraints stop vortex lines being stretched or twisted. Physically this upscale energy transfer occurs by merging of vortices and leads to the production of coherent structures in the flow. This scenario is an appropriate model for geophysical flows which are known to contain very energetic vortices mesoscale oceanic eddies and atmospheric highs and lows. This upscale transfer of energy is inhibited at the Rossby deformation radius:

$$R_D = \frac{N}{f} h . \qquad (5)$$

where h is the characteristic scale of the depth of the thermocline and f the Coriolis parameter , $f = 2\,\Omega\, sen\, \varphi$. The energy limitation is caused by baroclinic instability at larger scales, which accounts for the dominant observed size of geophysical vortices.

4.2- Velocity measurements from coastal Radar

WERA HF radar technology was recently used in the eastern part of the Gulf of Lions (Reffray et al 2004, Schaeffer et al. 2010) coordinating resources from UPC and Toulon university that allowed to point out the occurrence of an anticyclone eddy confined on the area near the Rhone estuary at the narrow shelf. Such a dominant vortex feature was observed several times during winters 2005 and 2006 with a persistence of several days (i.e. Fig. 3).

Process oriented modeling exercises allowed to relate the eddy formation to the wind forcing and local complex bathymetry. Different mechanisms have been described, able to induce the occurrence of such mesoscale eddies as related either to the instability of the Northern shelf edge current or to the wind forcing (Schaeffer et al 2010). In both cases, bathymetry and the Rhone river plume front play a dominant role in the confinement of the eddy. In particular, inertial motion due to wind reversal is often responsible of anticyclonic vortex formation in the water column which can be masked at sea surface by an upwelling motion. This mechanism is also believed to act in the Ebro river plume area as discussed in(Carrillo et al 2001, 2008), and may be seen in figure 1.

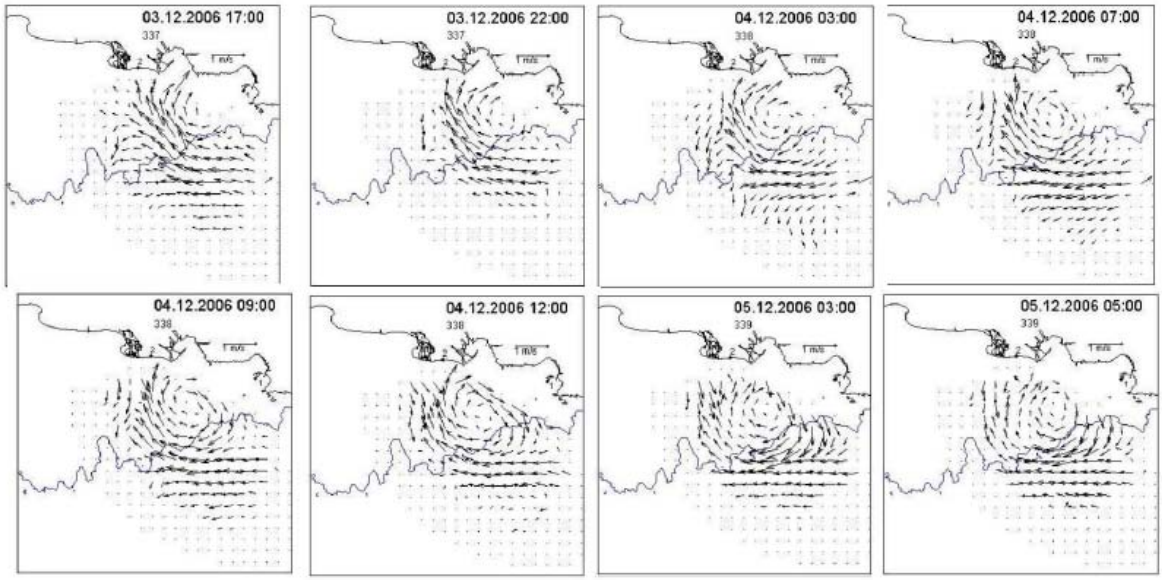

Fig. 2. A sequence of vortex formation during winter 2006

Integrated ROFI measurements and satellite observations have been used to simulate numerically the velocity patterns under similar conditions on selected dates Moreover, a previous investigation (Reffray et al, 2004) using a factor analysis related eddy formation was able to model the vorticity in the area of the Rhône river plume due to non linear processes. Wind forcing, Liguro Provenzal shelf current and river plume fronts influence were investigated separately and in the end, the non linear effects and the modeled turbulent diffusivity were found to be the same order of magnitude in the location of eddy formation. This was also confirmed from satellite image information and from coastal radar measurements.(Figure 4) As shown also in the work of Carrillo et al (2008) the river plume flow is one of the key elements generating certain features as long lived eddies and affecting the local diffusivity in the area.

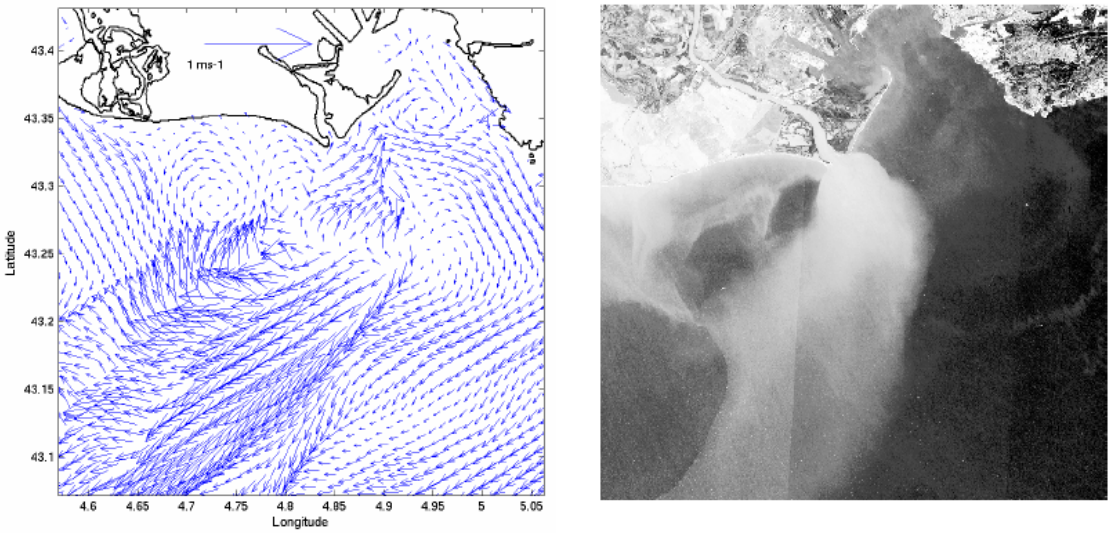

Fig 3. Vortex formation in the Rhone river Freshwater influence region during high river flow

Several eddies were observed on TSM SPOT images in northerly land out-shore wind condition able to reconnect the re suspended coastal waters to the river plume, enabling offshore transport of ancient deposit in the pro-delta. In this case, secondary counter rotating vortices play also a very important role in local mixing and transport. Numerical coastal model forced by idealised northerly wind are in a surprising good agreement with observations.

In the complex and varying distribution vortices in the ocean, local shear will transform slicks in the surface to align and follow the local flow so the resulting pattern tends to be spiral as shown by (Munk, 2001). The mixing processes at large scale produce *stirring*, which maintains large gradients of the tracers. But in order to mix at molecular level in an irreversible fashion, the energy has to cascade to the smallest internal scales (Kolmogorov or Batchelor scales). In time the area where diffusion takes place increases and the variation of area in time may be used as a measure of the overall diffusion coefficient, what has been noticed in time sequence of the eddy distribution is the occasional energetic burst that in a couple of days destroys the existing eddy distribution, after these sudden meteorological or hydraulic driven intermittent forcing, the eddies tend to grow and decay. Thermal images of the ocean, coupled with clorophyl-colour ones are suited to describe the different forcing and time evolution, but the effect of the cloud cover precisely hampers the study of the most active forcing periods. SAR images on the other hand are not affected by clouds or bad weather and the obtained eddy statistics have no weather related bias. In figure 4 a secuence of the dispersion of an oil spill (tracers in a turbulent KS flow) is shown as an example, different sites with local conditions may be modeled in a Lagranian statistical/stochastic way

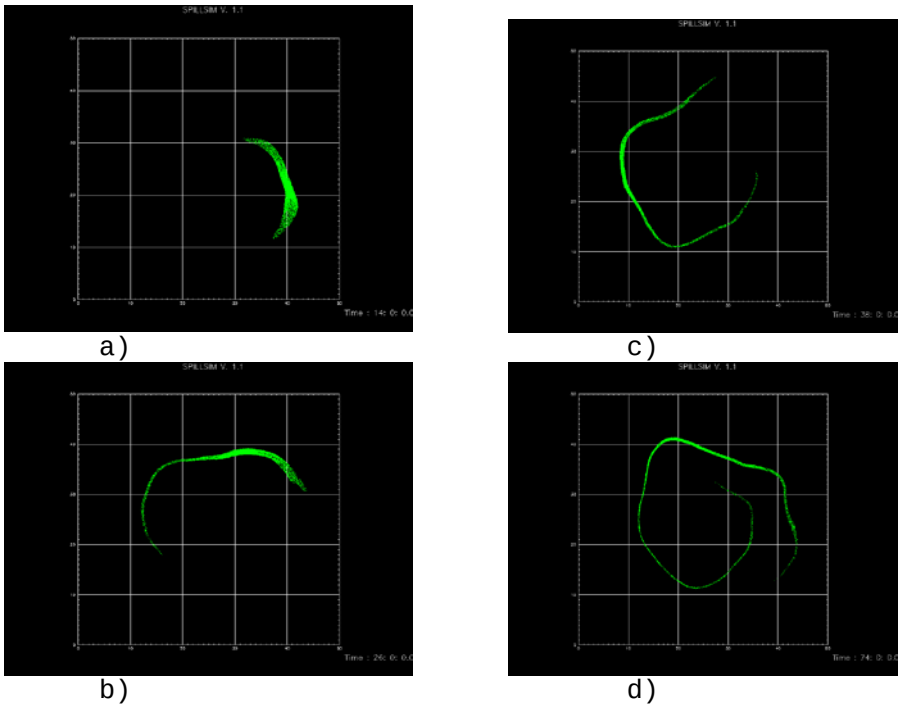

figure 4: Secuence a-d) of the dispersion of an oil spill in a turbulent KS flow

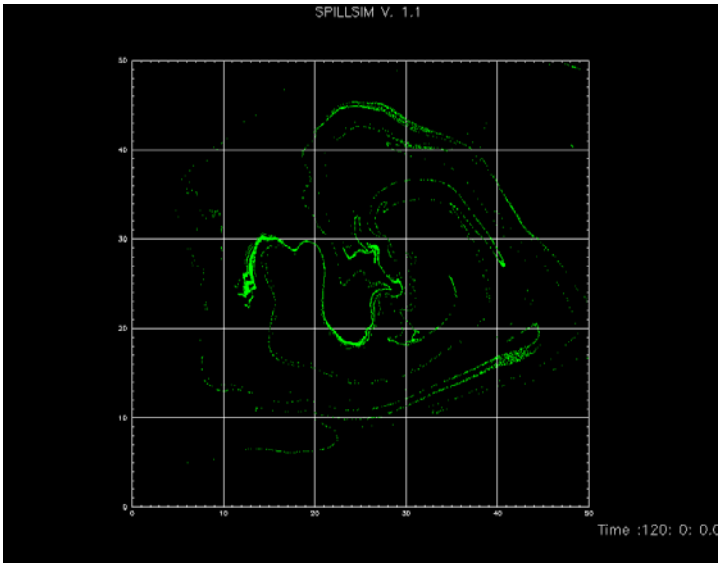

Figure 5: A weathered oil spill after a long time

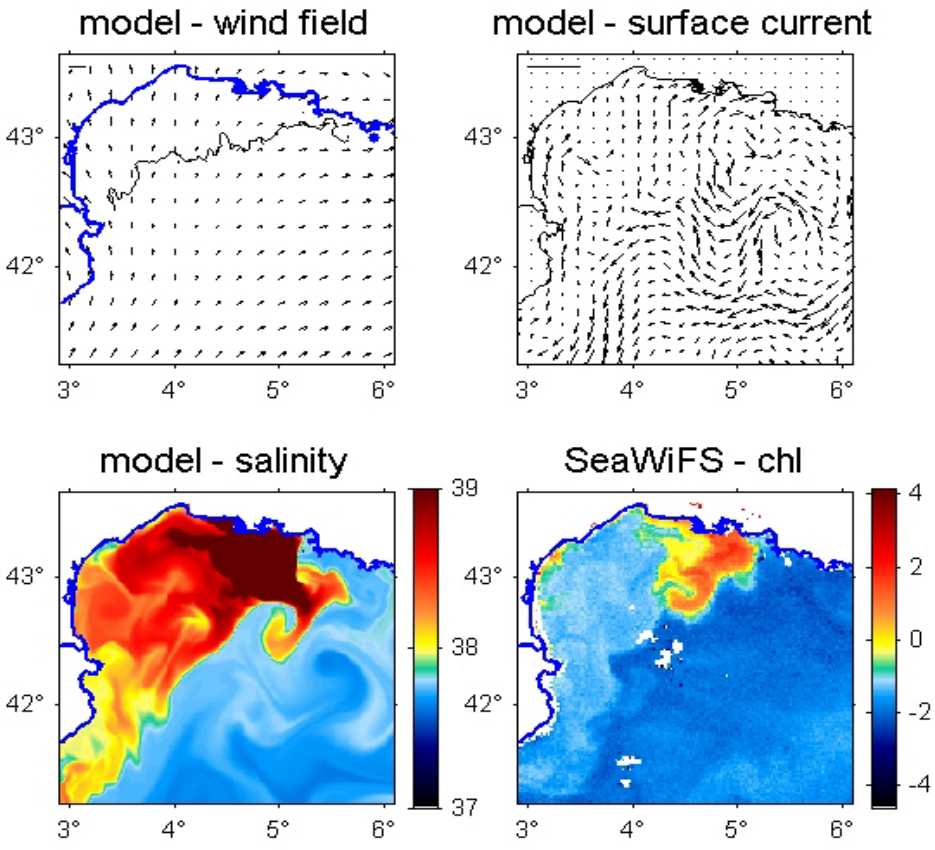

Figure 6: Comparison of salinity and clorophile with models of the gulf of Lions

5- Discussion and conclusions

Many features have been identified with structures and phenomena observed in several experiments, and understanding of atmospheric and ocean dynamics has been significantly advanced being able to compare different features marking the ocean surface to its velocity dynamics (Figure 6). The experiments and observations have provided new insights about the dynamics and have revealed a wide range of nonlinear behaviour. When the instability is caused by differential heating or by buoyancy there seems to be a range of very different dynamic regimes detected in the experiments, but not identified in the ocean. Work by (Carrillo et al., 2001, 2008) has revealed the complex interactions possible between lateral (or coastal) stirring and the rotating-stratified flow dynamics. The investigation of such strongly non-homogeneous flow that leads to intermittent two dimensional turbulence is believed very important if correct parametrizations of pollutant dispersion *(such as Oil spills*) in coastal areas are to be made.

The availability of a large scale flow allows both to measure Eulerian velocities with precision as well as Lagrangian flows using particle tracking as well as local measurements of diffusivity by video recording the dispersion of neutral tracers. A possible oil spill prediction technique, involves the releasing of hundreds of small and inexpensive tracer (GPS) Lagrangian buoys near an accident to aid the predictions of coastal currents (Bracco et al 2004, Redondo and Platonov 2009). Diffusion in the ocean exists on different space scales: from a molecular level (molecular viscosity and diffusion) to oceanic turbulent processes (from centimetres and metres vertical scale to tens and hundreds of kilometres of horizontal scale).

After an initial Balistic phase, the non-lineal processes begin to dominant and new dynamic scales appear, the maximum concentrations of the pollutions C_{max} in the centre of surface patches on the Baltic Sea and in the Black Sea showed that C_{max} is proportional to t^{-3}. The study of the turbulent diffusion is based on two methods. The first is Lagrangian (monitoring and numerical analysis of the motion of the particles or tracers) and the second one is Eulerian with a characterization of the spatial distribution of the velocities, correlations and energy spectral characteristics that may affect locally the turbulent diffusion.

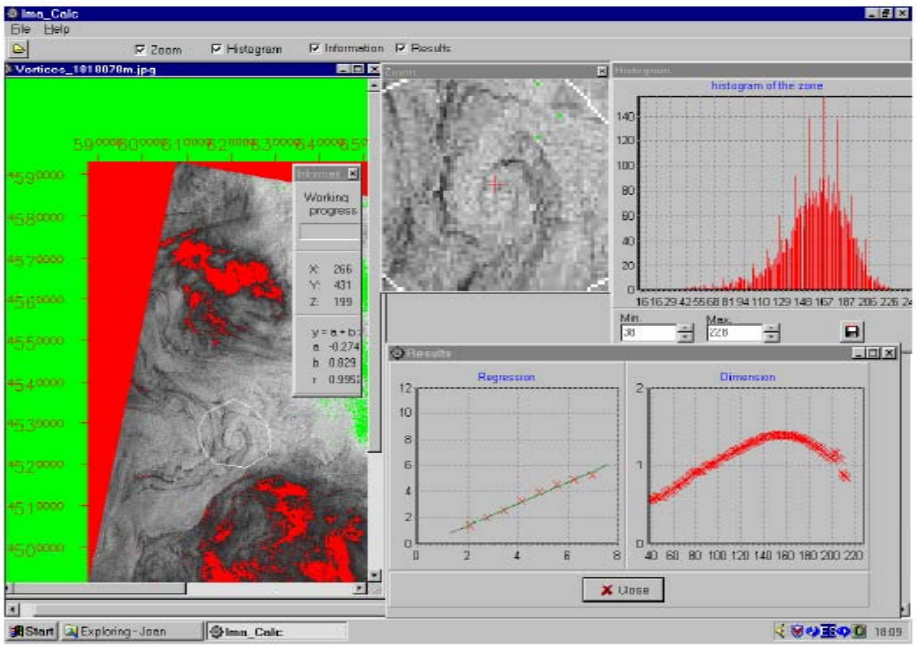

Figure 7: Fractal analysis of an eddy i the ocean detected by SAR

Using a systematic analysis of satellite images, there is a method of calculation of the average eddy diffusivity from a sequence of SAR images, using dimensional analysis and the local scales measured as integrals of the SAR reflectivity spatial correlations, here the local influences of the wind and the currents are important. Nevertheless, on the long run, horizontal directions will average out so using a single integral length scale defined in will be enough together with the inertial frequency. The method involving the multi-fractal dimension measurements is much more elaborated and seems to have a better theoretical justification, in the sense that it is possible that different concentrations showing different fractal dimensions may be due to different levels of intermittency and thus different spectra, which are not necessarily inertial nor in equilibrium. The specific multifractal spectra of the eddies in the ocean as shown in figure 7 can also be use to distinguish different types of traces in the ocean surface.

We should be able to relate spatial topological features detected by SAR to the local diffusivity *K*, which depends on Waves, wind and local bathimetry as shown by Bezerra et al.,(1998).

In the apendix the relation between the fractal dimension and the diffusivity may is derived from basic spectral and dimensional analysis (Diez et al 2008).

Then reference plots of features such as the maximum fractal dimension with the integral of the fractal dimension over all possible intensity levels of SAR can be used to predict the behaviour of the oil spills. The topological structure may also help us to distinguish between oil seeps from the ocean bottom (more distributed) and oil spills from ships (elongated). (Redondo and Platonov 2009).

The distribution of meso-scale vortices of size the Rossby deformation scale and other dominant features. Multi-fractal analysis can be used to distinguish features in the ocean surface. The SAR images exhibited a large variation of natural features produced by winds, internal waves, the

bathymetric distribution, by convection, rain, etc as all of these produce variations in the sea surface roughness so that the topological changes may be studied and classified. In a similar way topography may be studied with the methodology described An additional unique value is obtained that characterizes the overall spatial fractal dimension of the system integrating the multifractal functions. Several polarizations of the SAR exhibit their different structure functions up to 6th order. The flatness or Kurthosis is a statistic parameter which indicates the shape of the pdfs of the SAR intensity, and seems to be a very good indicator of the degree of existing structure; when flatness changes with scale following a potencial law, intermittency is present. Both the multifractal spectra and the distribution of the Flatness function F are found to be useful to measure intermittency, when it is applied to the correlations between the different SAR polarizations. Comparisons with the standard multi-fractal formalism also may reveal the importance of anisotropy.

The authors thank the ENV4-CT96-0334 European Union Project, INTAS, ISTC1481, and ESA-IP2240 as well as MEC projects SB2000-0076 and ESP2005-07551.

Apendix

Statistical and spectral analysis of Fractal Vortices

In the process of the turbulent eddy three dimensional Kolmogorov type cascade we my assume that the basic hypothesis of Kolmogorov(1942, 1962) apply, then a) the properties of the eddy hierarchy at a certain scale only depend on the local dissipation at that size L_L, at scale $\underline{L}$ and b) that there also exists an intermediate "inertial range" of eddies $Lo >> L >> L_k$ the viscosity does not play any role. In this range, all the properties should be determined by dissipation, at least over a range of scales, but keeping a less restrictive homogeneous and locality vievs with respect to intermittency)

We can relate how the characteristic speed $v(L)$ for eddies of size L depends on L. Since the dimensions of viscosity are velocity^2/time, the only dimensional combination of dissipation and L yielding a dimensional estimation of:

$$v(L) \approx (\varepsilon L)^{1/3}$$

And thus the fractional distribution of the energy per unit mass and length will be

$$E(L)(dL/L) \approx v(L)^2 (dL/L) \approx (\varepsilon L)^{2/3} (dL/L) \approx \varepsilon^{2/3} L^{-1/3} dL$$

That leads to the traditional Kolmogorov spectral 5/3 law

$$E(k)dk \approx \varepsilon^{2/3} k^{-5/3} dk$$

It is of crucia importance to study the linfluence of complex turbulence structure on diffusion, so if we assume that a scaling exponent exists, the moments of the relative distance *R* between a pair of particles scale with time in the limit of long times. As a consequence, the mean value of the square of the displacement *R* as a function of time in the long-time limit is given by

$$\left\langle R^2 \right\rangle \approx t^{2/(1-\zeta_1)}$$

Which is larger than the classical brownian type of diffusion, and also as suggested by Richardson as early as 1925, with the scaling exponent 1/3 associated to Hurst scaling exponent.

$$\left\langle R^2 \right\rangle \approx t^3$$

The fact that dissipation is not constant as assumed in the Kolmogorov K41 theory and that it also scales spatially leads to a more general K62 definition of the scaling exponents of the structure functions so that now $\zeta_p = f(h,p)$ and not just $\zeta_p = hp=p/3$ (Frisch 1975).

$$\left\langle v(L)^p \right\rangle \approx L^{p/3} \left\langle \varepsilon(L)^{p/3} \right\rangle \approx L^{\zeta_p}$$

One of the first theories seeking to predict the structure function exponents was in fact given by Kolmogorov in K62, also acknowledging the contribution of Obukhov, this is known as the log normal theory assuming that this is how dissipation probability distribution functions behave, i.e.

$$\Pr[\ln \varepsilon(x)] \approx \exp\left[-\frac{\left(\ln \varepsilon(x) - \langle \ln \varepsilon(x) \rangle^2 \right)}{2\mu} \right]$$

And this new constant that describes the intensity of the fluctuations of the dissipation in a logarithmic time scale is what is presently known as the intermittency parameter, this parameter has been measured for homogeneous, stationary and isotropic turbulence giving values in the range 0.2 to 0.6, but for real turbulence the situations seems much more complicated as discussed by Mahjoub et al (1998)

$$\left\langle \partial\varepsilon(r) \partial\varepsilon(r+L) \right\rangle \approx \left\langle \varepsilon^2 \right\rangle (L/L_0)^{-\mu}$$

With this theory, the scaling exponents would depend both on p and the intermittency parameter, so the expression for the p^{th} order velocity structure function will be:

$$\left\langle v(L)^p \right\rangle \approx L^{p/3} L^{\mu p(3-p)/18}$$

And the expression for the energy spectra as a function of the wave-number modifies the non-intermittent K41 3D cascade given by the5/3 law as:

$$E(k) \approx \varepsilon^{2/3} k^{-5/3-\mu/9}$$

In a similar way, now the average area of a spill or a passive tracer cloud would be also modified by intermittency

$$\left\langle R^2(t)\right\rangle \approx t^{51/(18-\mu)}$$

The general expression for the structure function scaling exponents is then

$$\varsigma_p = \frac{p}{3} + \frac{\mu}{18} p(3-p)$$

If we use the relationship between the fractal dimensión (or more exactly the máximum of the multifractal measures that describe the contours of dissipation), we may relate it with the intermittency following the argument for a stratified flow of Redondo(1990) or Diez et al.(2008):

$$L(n) = L_0 2^{-n}$$

And from the fractal dimensión definition:

$$N(n) = N_0 2^{Dn}$$

The volumen at the nth generation step of an intermittent vortex cascade is

$$V(n) = N(n)L(n)^3 \approx N_0 {L_0}^3 2^{-(3-D)n}$$

With a volumen ratio of:

$$\beta(n) = V(n)/V_0 = 2^{-(3-D)n} = \left[L(n)/L_0\right]^{3-D}$$

And dissipation, velocity, Energy and spectral energy given by:

$$\varepsilon \approx \beta(n) v^3(n) \tau^{-1}(n)$$

$$v(n) \approx \left[\varepsilon L(n) \beta^{-1}(n)\right]^{1/3}$$

$$E(n) \approx \beta(n) \varepsilon^{2/3} \left[L(n)/\beta(n)\right]^{2/3}$$

$$E(k)dk \approx \varepsilon^{2/3} k^{-5/3} \left[kL_0\right]^{-(3-D)/3} dk$$

$$\left\langle v(L)^p \right\rangle \approx \varepsilon^{p/3} L^{\varsigma_p}$$

And thus we obtain what is often called the multifractal (or beta) method Frish(1995)

$$\varsigma_p = \frac{p}{3} + (3-p)\frac{(3-D)}{3}$$

If we want to estímate diffusion in a fractal environment, we may substitute the first order structure function scaling exponent (p=1) as

$$\varsigma_1 = \frac{1}{3} + (3-1)\frac{(3-D)}{3} = \tfrac{1}{3}(7-2D)$$

Relating the fractal dimension to structure function scaling exponents.

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