

# OPTIMAL ENERGY CONSERVING DISCONTINUOUS GALERKIN METHODS FOR THE WAVE PROPAGATION PROBLEMS IN HETEROGENEOUS MEDIA

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Solving wave propagation problems within heterogeneous media has been of great interest and has a wide range of applications in physics and engineering. The design of numerical methods for such general wave propagation problems is challenging because the energy conserving property has to be incorporated in the numerical algorithms in order to minimize the phase or shape errors after long time integration. In this talk, we will discuss multi-dimensional wave problems and consider linear second-order wave equation in heterogeneous media. We will present an LDG method, in which numerical fluxes are carefully designed to maintain the energy preserving property and accuracy. We propose compatible high order energy conserving time integrators and prove the optimal error estimates and the energy conserving property for the semi-discrete methods. Our numerical experiments demonstrate optimal rates of convergence, and show that the errors of the numerical solution do not grow significantly in time due to the energy conserving property.

## REFERENCES

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