11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

TOWARDS AN EFFICIENT NON-INTRUSIVE POLYNOMIAL CHAOS APPROACH FOR HIGH DIMENSIONAL STOCHASTIC PROBLEMS USING A REDUCED BASIS APPROACH

Dinesh Kumar, Mehrdad Raisee and Chris Lacor

Fluid Mechanics and Thermodynamics Research Group, Department of Mechanical Engineering, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, BELGIUM
E-mail: dkumar@vub.ac.be; m_raisee@yahoo.com; chris.lacor@vub.ac.be

Key words: *Stochastic, Polynomial Chaos, Proper Orthogonal Decomposition, Model Reduction*

In the present paper, the results of ongoing work within the FP7 project UMRIDA will be presented and discussed. The aim of UMRIDA is to upgrade the TRL level of UQ in aeronautics to level 5-6. The main challenge industrial applications of UQ are facing, is the curse of dimensionality as a result of a large number of uncertainties. Within UMRIDA therefore different methodologies to deal with this will be investigated. Within VUB a reduced basis approach is being investigated. The main idea is to extract the optimal orthogonal basis via cheap calculations on a coarse mesh and then use them for the fine scale analysis. To validate the developed reduced-order model, the method is implemented to the stochastic 2D heat diffusion and stochastic incompressible boundary-layer. The results show that the developed non-intrusive model reduction scheme is able to produce satisfactory results for the statistical quantities. It is found that the memory requirements and the computation-time of the reduced-order model is lower than that of the standard PC. In the paper results obtained with numerical quadrature and regression schemes will be presented.

Following our previous work on UQ at VUB, here we employed a Polynomial Chaos (PC) approach to model uncertainty propagation. Developing efficient reduced-order models for reducing the computational cost associated with the stochastic analysis is of great interest for the prediction of complex industrial flows with large number of uncertain parameters. In the framework of UMRIDA project here we focus on efficient reduced order models for uncertainty quantification. In recent years, several model reduction techniques have been proposed for uncertainty quantification. One informative example of such works is Doostan et al. [2007] who proposed an intrusive model reduction technique for chaos representation of a SPDE to tackle the curse of dimensionality. They applied it successfully to a stochastic 2D solid mechanics problem. Here the methodology is extended to non-intrusive PC.

In classical PC an unknown stochastic field, say u, is decomposed in a basis of polynomials (preferably according to the Askey scheme):

$$u(x, y; \xi) = \sum_{i=0}^{P} u_i(x, y).\psi_i(\xi) \tag{1}$$

where the total number of terms are P +1 = (p + ns)!/p!ns! and grows very fast with increasing stochastic dimension (ns) and polynomial chaos order (p). In a reduced basis approach the number of terms can be reduced significantly. An optimal basis can be found from a POD of the field (also known as Karhunen-Loeve expansion) but this requires the knowledge of the covariance of the (unknown) field. The main idea is to obtain this covariance, and hence the reduced basis, from a coarse grid simulation, hereby assuming that the stochastic behavior is not very sensitive to the spatial accuracy. The resulting expansion is now of the form:

$$u(x, y; \xi) = \sum_{i=0}^{m} u_i(x, y).z_i(\xi) \tag{2}$$

where m is very small, typically less than 10, and the z_i polynomials are known from the POD analysis on the coarse grid. The calculation of the u_i, on the fine grid level, can now be obtained as in standard non-intrusive PC approaches, e.g. based on numerical quadrature or on regression.

REFERENCES

[1] Doostan, A., Ghanem, R., and Red-Horse, J. (2007). Stochastic model reduction for chaos representations. Computer Methods in Applied Mechanics and Engineering, 196:3951-3966.

[2] Hosder, S., Walters, R. W. and Balch, M., Efficient Sampling for Non-Intrusive Polynomial Chaos Applications with Multiple Uncertain Input Variables, 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA 2007-1939, 15th 23-26 April 2007, Honolulu, Hawaii.