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A NEW METHOD FOR SCATTERING PROBLEMS IN UNBOUNDED ANISOTROPIC ELASTIC MEDIA

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Non destructive testing (NDT) methods using guided waves are currently developed to control the integrity of elastic plates, often made of composite materials, arising for instance in nuclear plants or aircrafts. Efficient and accurate numerical methods of simulation of such experiments are useful, as they help to design optimally the NDT techniques. Among the different numerical tools required [1], we are concerned here by the simulation of the scattering by the defect.

More precisely, our objective is to develop a method to simulate the diffraction of a time harmonic guided wave by a bounded arbitrary defect (crack of arbitrary shape, corrosion zone etc...) in an infinite anisotropic elastic plate. The difficulty is to find a way to restrict the finite element computation to a small box containing the defect. Indeed classical methods fail in this case. In particular, it is well known [2] that Perfectly Matched Layers do not work in anisotropic elastic media because outgoing waves may become unstable in the layer, and no general remedy has been proposed up to now for this. Let us mention that Perfectly Matched Layers do not work neither in (even isotropic) 2D plates, due to the existence of backward propagating waves [3]. The configuration of interest here combines the both difficulties.

Our idea is inspired by the work of Patrick Joly and Sonia Fliss for periodic media [4]. Let us present the method on the simpler case of a bounded defect in an infinite 2D anisotropic elastic medium (transverse resonance is not taken into account).

The whole domain is partitioned into five subdomains:

- a square that surrounds the defect in which a classical finite element representation of the solution is used,

- and four half-spaces, parallel to the four edges of the square, in which analytical representations of the solution are derived: indeed, by using a partial Fourier transform, then solving explicitly the differential equations obtained and finally selecting the “outgoing” solution, the field in each half-space is expressed as a function of its trace on the boundary, which is itself discretized using 1D discontinuous finite elements (P0).

The different unknowns (the 2D finite element unknown in the square and the four 1D finite element traces on the boundaries of the half-spaces) are finally coupled by well-chosen transmission relations which ensure the compatibility between the five representations, in the areas where the subdomains overlap.

The method has been validated successfully in the case of 2D isotropic and anisotropic acoustic media, and then extended to 2D elasticity.

The mathematical properties of the formulation and the efficiency of the method strongly depend on the presence or not of overlaps between the finite element box and the four half-planes.

Assuming a slightly dissipative medium, we proved that the formulation with overlaps has good Fredholm properties, which ensures a rapid convergence of a GMRES algorithm. On the other hand, the well-posedness of the multi-domain formulation for all frequencies is only proved for the formulation without overlaps.

REFERENCES

[1] L. Taupin, A. Lhémery, V. Baronian and A.-S. Bonnet-Ben Dhia, *Scattering of obliquely incident guided waves by a stiffener bonded to a plate*, Journal of Physics: Conference Series, 353, 2012.

[2] E. Bécache, S. Fauqueux and P. Joly, *Stability of perfectly matched layers, group velocities and anisotropic waves*, Journal of Computational Physics, 188, 2003.

[3] A.-S. Bonnet-Ben Dhia, C. Chambeyron and G. Legendre, *On the use of perfectly matched layers in the presence of long or backward propagating guided elastic waves*, Wave Motion, Available online 2013.

[4] S. Fliss and P. Joly, *Exact boundary conditions for time-harmonic wave propagation in locally perturbed periodic media*, Applied Numerical Mathematics, 59, 2009.