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USING THE STOCHASTIC POISSON-BOLTZMANN EQUATION TO QUANTIFY NOISE IN NANOWIRE BIO- AND GAS SENSORS

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Abstract. In recent years, nanowire field-effect bio- and gas sensors have been shown experimentally to detect minute concentrations of biomolecules such as DNA oligomers and tumor markers and of toxic gas molecules such as carbon monoxide. The fully self-consistent modeling and simulation of these sensors poses several theoretic and computational problems (see, e.g., [1–6]).

Here we model the randomness of the molecules in these sensors using the stochastic nonlinear Poisson-Boltzmann equation

$$-\nabla \cdot (A(x,\omega)\nabla u(x,\omega)) + \kappa(x,\omega)\sinh(u(x,\omega)) = f(x,\omega).$$

The stochastic location, stochastic orientation, and stochastic charge of the molecules gives rise to stochasticity in the electrostatic potential u and hence in the current through the nanowire sensor [1,6]. In addition to the mean values yielded by deterministic models, the variance (and standard deviation) is calculated here in order to find signal-to-noise ratios (see Figure 1).

We report on the advantages and disadvantages of various numerical approaches, namely Monte Carlo, quasi Monte Carlo, stochastic collocation, and stochastic Galerkin. The challenge here is to efficiently approximate the many stochastic dimensions.

Finally, the graded-channel approximation yields an approximation to the current through the transducer. We study the effect of device parameters on the signal-to-noise ratio of the sensor. The simulated current-voltage characteristics agree very well with measurements.

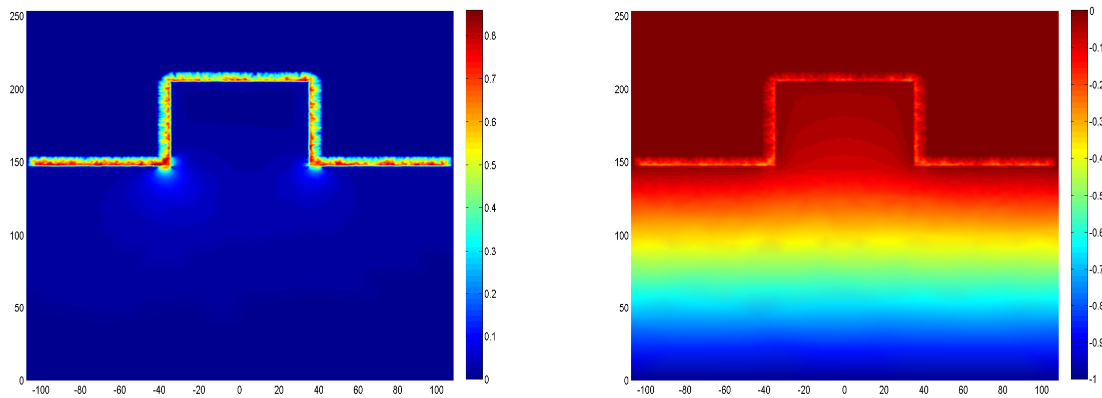

Figure 1: Standard deviation (left) and expected value (right) of the electrostatic potential in the cross section of nanowire field-effect biosensor.

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