11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

ON THE NITSCHE AND THE SHIFTED PENALTY METHOD

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Key Words: *Contact Mechanics, Penalty Method, Nitsche Method, Augmentation.*

The Nitsche method [1], originally developed for matching of different meshes, in more recent years has been adopted also for enforcing boundary conditions [2]. Moreover, it has been used within domain decomposition methods with non-matching grids in [3] and [4], and it has had a noticeable success also in contact mechanics both for frictionless [5] and frictional [6] formulations. The method is characterized by a stress vector at the interface that is computed from the stress field of the bodies. Due to the presence of negative terms in the diagonal of the resulting equation system, a penalty-like term has to be added for stabilization. However, the solution does not depend on the penalty parameter as strongly as in the pure penalty approach. In summary, the Nitsche method presents some interesting features that can reduce the well-known shortcomings of the penalty approach.

A variation of the classical penalty method that is also characterized by a reduction of the negative features of the original formulation has been recently proposed in [7]. In this case the crucial modification concerns the introduction of a shift parameter that moves the penetration toward zero without any penalty increase. The shift term, s, modifies the classical penalty contribution as follows

$$\frac{1}{2}\epsilon g_n^2 \quad \rightarrow \quad \frac{1}{2}\epsilon(s+g_n)^2$$

However, the shift does not act as a new unknown, like in the Lagrangian Multiplier method. The shift update works like a sort of augmentation, which is performed at each Newton's iteration. Its update is simply and very rapidly computed enforcing the condition that the solution has to take place at $g_n = 0$. The numerical examples presented in [7] have shown interesting characteristics, like e.g. superlinear convergence rate and exact constraints enforcement. The basic idea can be easily evidenced considering the simple test case of Fig. 1, which gives the following potentials that have to be minimized when the gap is closed

$$\frac{1}{2}Ku^2 - Fu + \frac{1}{2}\epsilon g_n^2 \qquad \text{Penalty}$$
$$\frac{1}{2}Ku^2 - Fu + \frac{1}{2}\epsilon(s+g_n)^2 \qquad \text{Shifted penalty}$$

Figure 1: Simple test case.

The different ways to move the solution toward the exact one is shown, respectively, in Fig. 2 and Fig. 3. The comparison of the Figures outlines immediately two remarkable advantages of the shifted penalty method, i.e. the possibility to reduce the penetration without increasing the penalty parameter, and the capability go get the exact solution.

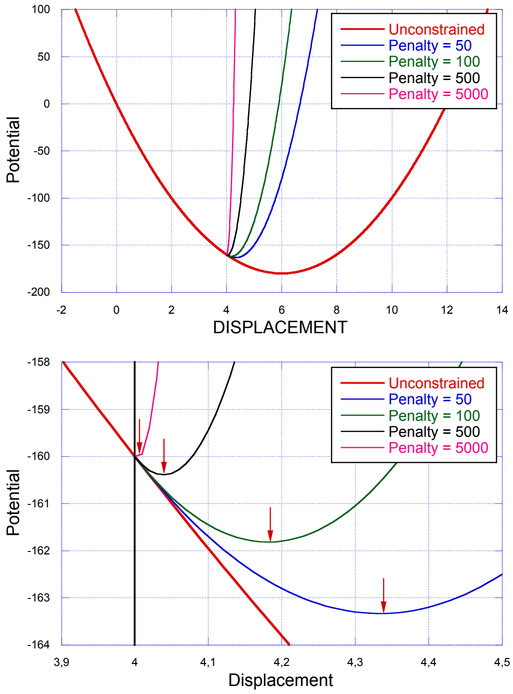

Figure 2: Potential modification with increasing the penalty parameter.

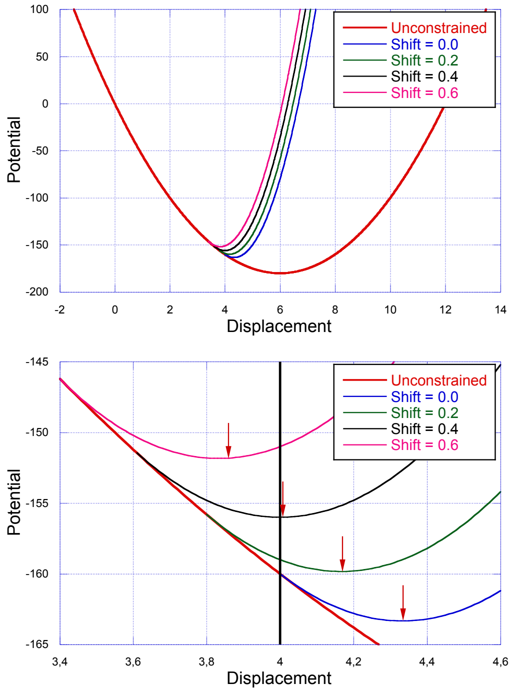

Figure 3: Potential modification with increasing the shift parameter.

The method has been formulated and tested in the classical context of frictionless contact mechanics. However from the theoretical point of view it can be used also in presence of friction and for the original scope of the Nitsche Method, i.e. for matching of different meshes. In this paper this aspect is widely explored and tested.

REFERENCES

[1] J. Nitsche, Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind. *Abhandlungen in der Mathematik an der Universität Hamburg*, Vol. **36**, pp. 9–15, 1970.

[2] R. Stenberg, On some techniques for approximating boundary conditions in the finite element method. *J. Comput. Appl. Math.*, Vol. **63**(1-3), pp. 139–148, 1995.

[3] R. Becker, P. Hansbo, A finite element method for domain decomposition with non-matching grids. *Technical Report No. 3613*, INRIA, Sophia Antinopolis, 1999.

[4] R. Becker, P. Hansbo, R. Stenberg, A finite element method for domain decomposition with non-matching grids. *Math. Model. Numer. Anal.*, Vol. **37**(2), pp. 209-225, 2003.

[5] P. Wriggers, G. Zavarise, A formulation for frictionless contact problems using a weak form introduced by Nitsche. *Comput. Mech.*, Vol. **41**(3), pp. 407-420, 2008.

[6] F. Chouly, An adaptation of Nitsche's method to the Tresca friction problem. *J. Math. Anal. Appl.*, Vol. 411(1), pp. 329-339, 2013.

[7] G. Zavarise, The shifted penalty method, *Comput. Mech.*, submitted.