

## New insights into viscoelastic contact mechanics between rough solids

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A large variety of components of engineering interest, including for example tires, dampers and seals, employs rubbers and rubber-based composites showing a marked viscoelastic behavior. Due to the complexity intrinsically involved to simulate the mechanics of these materials, the scientific community has developed a wide number of contributions (see for example [1-4]). Here, particular attention is paid to investigate the steady state sliding contact between viscoelastic rough solids. Indeed, the authors have already proposed in [2] a novel methodology to determine the contact solution for the steady-state contact between smooth bodies. For this last case, the numerical theory has also been experimentally validated in [2]. In this paper, we show how the same method can be effectively employed when accounting for the role played by the surface roughness. This is not trivial at all since it requires to take into account a number of scales covering several orders of magnitude and, therefore, generally speaking, to have a very fine discretization of the space and the time domains. On the other hand, modern engineering design must consider roughness since it influences phenomena of paramount importance, like viscoelastic friction and percolation.

Hence the necessity of investigating the main peculiarities of viscoelastic rough contact.

Now, regarding the numerical methodology, the viscoelastic contact of two sliding surfaces is formulated in terms of a Green's function approach, thus extending the innovative boundary element formulation proposed in Ref. [4] for elastic materials. This procedure when compared to conventional FE methods [2] presents a noteworthy computational efficiency and an outstanding accuracy in the description of the interfacial stresses and strains. To this end, in the framework of linear viscoelasticity, we introduce the formulation based on the following integral equation:

$$u(\mathbf{x}) = \int_A d^2x' G(\mathbf{x} - \mathbf{x}', \mathbf{v}) \sigma(\mathbf{x}') \quad (1)$$

Where  $\mathbf{x} = (x, y)$  is the in-plane position vector,  $u(\mathbf{x})$  is the surface displacement,  $\sigma(\mathbf{x})$  is the interfacial stress in the contact area  $A$ ,  $\mathbf{v}$  is the sliding or rolling velocity, and  $G(\mathbf{x}, \mathbf{v})$  is the Green's function which parametrically depends on  $\mathbf{v}$ . The function  $G(\mathbf{x}, \mathbf{v})$  has been calculated by solving the problem of a viscoelastic material loaded with a moving (at constant velocity  $\mathbf{v}$ ) concentrated unit load. One of the main advantages of our approach is the capability of dealing with a very general linear viscoelastic material with a continuum distribution of relaxation times. Once calculated the stresses and the strains by inverting the linear system obtained discretizing Eq. (1), the viscoelastic friction force can be easily determined (see [2] for details).

The methodology so developed can be employed to elucidate the two main points marking the viscoelastic contacts: these are the viscoelastic friction and the anisotropy of the contact solution. As for the friction, we have the bell-shaped curve reported in Figure 1a: the curve, calculated for a paradigmatic one-relaxation-time material, shows zero friction in the elastic regimes and, in parallel, a maximum when the viscoelastic effects are greatest.

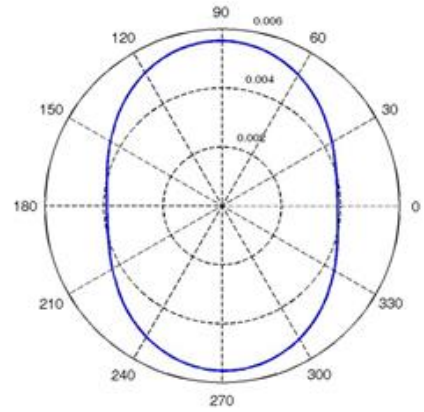
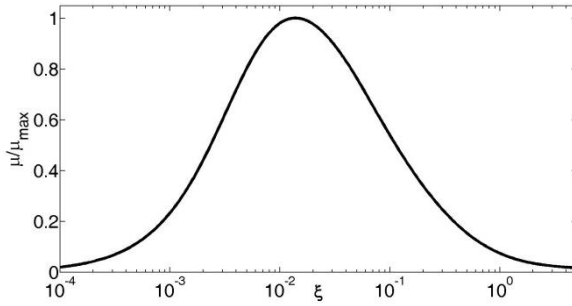


Fig. 1a: Friction coefficient as a function of the dimensionless sliding speed  $\xi = v\tau_0/L$  being  $v$  the speed,  $\tau_0$  the relaxation time and  $L$  the side of the computational cell. Fig 2b: Polar plots of  $m_2(\theta)$  for  $\xi = 1.17 \cdot 10^{-2}$ .

Furthermore, in the paper, we investigate the anisotropy of the contact solution in terms of contact area and of deformed region. This phenomenon is not deeply investigated in literature, but has an outstanding importance for example in all the components where fluid percolation and leakage have to be controlled. A possible quantification of the phenomenon can be carried out by looking at the quantity  $m_2(\theta)$  that is the second order spectral momentum of a profile trace made at an arbitrary angle  $\theta$  with respect to the x axis and can be calculated by the following relation:

$$m_2(\theta) = m_{20}\cos^2(\theta) + 2m_{11}\sin(\theta)\cos(\theta) + m_{02}\sin^2(\theta) \quad (7)$$

where  $m_{20}$  is the value of the profile second order momentum along the x axis ( i.e.  $\theta=0$  ),  $m_{02}$  is the value of the profile second order momentum along the x axis ( i.e.  $\theta=\pi/2$  ) and  $m_{11}$  is the association-variance of slope in these two directions. As a matter of fact, if the surface is perfectly isotropic, the momenta  $m_{20}$  and  $m_{02}$  are equals, while  $m_{11}$  is zero. In this case, therefore, by plotting  $m_2(\theta)$  in a polar diagram, we obtain a circumference with radius  $r=m_{20}=m_{02}$  . On the other side, if anisotropy is present, we have an ellipse, where the ratio between the axis, named  $\gamma$  , and the angle  $\Theta$  of the profile with the maximum  $m_2(\theta)$  can be employed as quantitative parameters. Indeed, in Figure 1b, we observe that the surface shows a consistent anisotropy with  $\gamma=0.70$  . Furthermore,  $\Theta$  is almost equal to  $\pi/2$  , thus being perpendicular to the sliding speed assumed parallel to x axis. Interestingly, we find the greatest anisotropy and the greatest fiction for the same speed value.

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