11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A RATE DEPENDENT MICROSTRUCTURAL CONSTITUTIVE MODEL OF INELASTIC EFFECTS IN SOFT FIBRED TISSUES

Estefanía Peña, Pablo Saez and Miguel A. Martínez

Applied Mechanics and Bioengineering. Aragón Institute of Engineering Research (I3A). University of Zaragoza. Spain
Centro de Investigación Biomédica en Red en Bioingeniería, Biomateriales y Nanomedicina (CIBER-BBN), Spain. fany@unizar.es

Key words: *Soft tissue, Microsphere, Viscous damage, Strain softening, Localization*

In this contribution a three-dimensional microsphere based constitutive model for anisotropic fibrous soft biological tissue is presented, including elastic anisotropy as well as inelastic effects (viscosity, softening, preconditioning and damage). The link between microstructural inelastic contribution of the collagen fibers and macroscopic response is achieved by means of computational homogenization, involving numerical integration over the surface of the unit sphere. In order to deal with the random distribution of the fibrils within the fiber, a von Mises probability function is incorporated, and the mechanical behavior of the fibrils is defined by an exponential-type model. The inelastic effects in soft biological tissues were modelled by internal variables that characterize the structural state of the material.

The microsphere approach tries to capture micro-structural information and transfer it into the macroscopic behavior via a homogenization scheme over the unit sphere $\mathbb{U}^2$. In this approach, $\mathbb{U}^2$ is discretized into m directions $\{\mathbf{r}^i\}_{i=1\ldots m}$ that are weighted by factors $\{w^i\}_{i=1\ldots m}$, with the consiton $\langle \mathbf{r} \rangle \approx \sum_{i=1}^m w^i \mathbf{r}^i = 0$ and $\langle \mathbf{r} \otimes \mathbf{r} \rangle \approx \sum_{i=1}^m w^i \mathbf{r}^i \otimes \mathbf{r}^i = \frac{1}{3}\mathbf{I}$. the anisotropic part of the SEDF is related to the fibers in the material. In a general situation with N families of fibers the anisotropic part of the SEDF can be expressed as

$$\Psi_{\text{ani}} = \sum_{j=1}^{N} \Psi_{\text{f}}^{j} = \sum_{j=1}^{N} \left[\frac{1}{4\pi} \int_{\mathbb{U}^2} n \rho_{\text{f}} \psi_{\text{f}} dA \right]_{\text{j}}, \tag{1}$$

where Ψ_{f}^{j} is the strain energy density function for the j-nth fiber family, n the chain density, ρ_{f} a statistical value associated with the fibrils dispersion and ψ_{f} the free energy density function of the fibril [1].

The free energy for the fibers is assumed to be of the form [3]

$$\Psi_{\text{ani}}\langle n\,\rho\,\psi_{\text{f}}(\lambda')\rangle \;=\; \frac{1}{4\,\pi}\int_{\mathbb{U}^2} n\,\rho\,(1-D_i)\psi_0(\overline{\lambda'}^i)\,\mathrm{d}A \approx \sum_{j=1}^{N}\left[\sum_{i=1}^{m} n\rho_i w^i[1-D_i]\psi^i_{j,0}(\overline{\lambda'}^i)\right] \quad (2)$$

with $D_i = D_i^\alpha + D_i^\beta$ the normalized scalars referred to as the damage, $D_i^\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $D_i^\beta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are monotonically increasing smooth functions with the following properties $D_i^\alpha(0) = 0$, $D_i^\beta(0) = 0$ and $D_i^\alpha + D_i^\beta \in [0,1]$ and $\psi_0(\overline{\lambda}^i)$ the effective strain energy density functions of each j family of fibers.

Some experimental results show that the amount of damage at a particular strain level presents rate sensitivity to the applied loading rate. Further, strain-softening and loss of strong ellipticity phenomena associated with damage mechanism impose numerical difficulties in finite element computations. To account the rate dependency and to regularize the localization problems, a viscous damage mechanism is used. Rate equations governing visco-damage behavior are obtained from their rate-dependent counterparts), by replacing the damage consistency parameter $\dot{\mu}_k$ by $\mu_k\bar{\Upsilon}(\Phi_k)$ for matrix and fibers respectively [2].

$$\dot{D}_k = \begin{cases} \mu_k\bar{\Upsilon}(\Phi_k)\bar{h}_k(\Xi_k, D_k) & \text{if } \Phi_k = 0 \text{ and } \mathbf{N}_k : \dot{\bar{\mathbf{C}}} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Here μ is the damage viscosity coefficient, $\bar{\Upsilon}(\Phi_k)$ denotes the viscous damage flow function and Φ_k. With this at hand, we write that $\dot{r}_{k_t} = \mu_k\bar{\Upsilon}(\Phi_k)$.

Acknowledgements. Support of the Spanish Ministry of Economy and Competitiveness (DPI2010-20746-C03-01 and the grant BES-2009-028593 to P. Sáez), as well as the support of the Instituto de Salud Carlos III through the CIBER are highly appreciated.

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