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CONTROLLING EUTROPHICATION IN A MOVING DOMAIN

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Eutrophication is an environmental process whereby large water bodies (lakes, estuaries, slow-moving streams, and so on) receive an excess of nutrients (nitrogen and/or phosphorus) that stimulate excessive undesirable plant growth (mainly, algae). This enhanced plant growth, usually known as an algal bloom, reduces dissolved oxygen in the water when dead plant material decomposes and can cause other organisms (fish, shellfish, seabirds, and even small mammals) to die, leading to changes in animal and plant populations and degradation of water and habitat quality. Nutrients come from many sources, such as fertilizers applied to agricultural fields, golf courses, and suburban lawns; deposition of nitrogen from the atmosphere; phosphate detergents; erosion of soil containing nutrients; and sewage treatment plant discharges.

In the present paper we are interested in controlling the eutrophication processes along a time interval $I = (0, T)$ inside a sensitive zone from a large water body $\Omega(t)$, where a wastewater outfall discharges polluted water with a high concentration of nutrients, coming, for instance, from a sewage treatment plant. In particular, we try to keep the level of eutrophication inside this zone $G(t) \subset \Omega(t)$ under safety thresholds, and with an economic cost (due to wastewater purification processes) as low as possible. From a mathematical viewpoint, the problem $(\mathcal{P})$ related to controlling eutrophication along a time interval I in a moving domain $\Omega(t)$ can be formulated as an optimal control problem with state and control constraints.

Eutrophication can be modelled by a system of partial differential equations, commonly presenting a high complexity due to the great variety of phenomena appearing on it. In this paper we have considered a simplified - but realistic - model, where only five biological species appear. So, we consider the variable $\mathbf{u} = (u^1, \ldots, u^5)$, where u^1 represents a generic nutrient concentration (usually nitrogen and/or phosphorus), u^2 the phytoplankton concentration, u^3 the zooplankton concentration, u^4 the organic detritus concentration, and u^5 the dissolved oxygen concentration. The evolution of these five species into a moving water domain $\Omega(t) \subset \mathbb{R}^3$ for a time interval I, can be described by the system of coupled nonlinear partial differential equations for advection-diffusion-reaction with Michaelis-Menten kinetics presented by the authors in the recent paper [1], where the moving domain problem has been analyzed from an ALE perspective. The source term corresponding to the wastewater outfall is modelled with a Dirac measure $g(t)\delta(x-b)$, where $g(t)$ represents the pollutant concentration (nitrogen and/or phosphorus in our case) discharged through the outfall, and b is the outfall location.

Moreover, we impose some technological constraints on the control g (related, for instance, to lower and upper bounds corresponding to the purification capacities of the sewage treatment plant: higher levels of purification lead to lower pollutant discharges). In this way, we assume that the control $g \in \mathcal{U}_{ad}$, a convex, closed, bounded subset of $L^2(0,T)$. We also impose several state constraints, aimed to guarantee the quality of water inside the sensitive zone $G(t)$ all along the time interval I. So, we need to assure that the averaged concentrations of the five species remain between some desired thresholds η^i and τ^i (respectively, the lower and upper bounds for species $i \in \{1,\ldots,5\}$ in the sensitive zone).

Finally, from a realistic viewpoint, we are interested in reducing the global economic cost of the purification process, i.e., minimizing the cost function $J(g) = \int_0^T m(g(t))dt$, where function $m(g)$ denotes the cost of the depuration in the treatment plant. Summarizing, the control and state constrained optimal control problem $(\mathcal{P})$ we are interested in consists of minimizing the cost function J such that the control g verifies the control constraints, and the state $\mathbf{u}$ satisfies the state constraints.

In a first part of the paper, we present several theoretical results on existence-regularity of optimal solutions, and their characterization by a first order optimality system. In the second part of the work a complete numerical algorithm for the resolution of the control problem is proposed, and several numerical results are also given. This numerical algorithm combines scientific software Freefem++ [2] (for the numerical resolution of the hydrodynamic equations and the state system) interfaced with interior point algorithm IPOPT [3] (for the resolution of the nonlinear constrained optimization problem, obtained from the space-time discretization of the continuous control problem).

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