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CONSTRUCTING OPTIMAL TRANSPORT MAPS FOR BAYESIAN INVERSE PROBLEMS

Youssef M. Marzouk, Alessio Spantini, and Tarek Moselhy

Department of Aeronautics and Astronautics
Massachusetts Institute of Technology, Cambridge, MA 02139, USA
email: ymarz@mit.edu

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We present a new approach to the Bayesian solution of inverse problems that entirely avoids Markov chain Monte Carlo simulation, by constructing a deterministic map that pushes forward the prior measure (or another reference measure) to the posterior measure. Existence and uniqueness of a suitable measure-preserving map is established by formulating the problem in the context of optimal transport theory. We discuss various means of computing the map efficiently through solution of a stochastic optimization problem; in particular, we use a sample average approximation approach that exploits gradient information from the likelihood function. The resulting scheme overcomes many computational bottlenecks associated with Markov chain Monte Carlo sampling. Advantages include analytical expressions for posterior moments, clear convergence criteria for posterior approximation, the ability to generate arbitrary numbers of independent samples, and automatic evaluation of the marginal likelihood to facilitate model comparison.

We then focus on methods for parameterizing and refining the map in order to take advantage of low-dimensional structure present in many inverse problems. While the posterior distribution may appear high-dimensional, the intrinsic dimensionality of an inverse problem is affected by prior information, limited data, and the smoothing properties of the forward operator. Often the change from prior to posterior can be well-approximated by a low-rank update, which has direct implications for the structure of the transport map. We will present the theoretically *optimal* update for linear inverse problems, and discuss iterative refinement techniques that extend this idea to nonlinear problems.