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LINEAR/NONLINEAR GEOMETRIC THERMOELASTIC RESPONSE OF STRUCTURES WITH UNCERTAIN THERMAL PROPERTIES

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The present investigation focuses on assessing the effects on the structural response of uncertainties in the thermal properties of a heated and loaded structure, i.e. in conduction and thermal capacitance, as well as the thermo-elastic coupling property, i.e. in thermal expansion coefficients. Based on a long series of successful validations, the maximum entropy nonparametric approach, e.g. see [1,2], will be chosen for the modeling of the uncertainty.

Such an analysis is best carried out within the framework of a nonlinear geometric model to account for possible events such as buckling and post-buckling behavior. Further, the reduced order modeling strategy of the temperature and displacements of [3] is adopted here. It is based on the representation of the temperature and displacements of the finite element nodes, stacked in the time varying vectors $\underline{T}(t)$ and $\underline{u}(t)$, in modal expansion forms, i.e.

$$\underline{T}(t) = \sum_{n=1}^{\mu} \tau_n(t)\, \underline{T}^{(n)} \quad \text{and} \quad \underline{u}(t) = \sum_{n=1}^{M} q_n(t)\, \underline{\psi}^{(n)} \qquad (1),(2)$$

In these equations, $\underline{T}^{(n)}$ and $\underline{\psi}^{(n)}$ are the thermal and structural basis functions, or modes, while $\tau_n(t)$ and $q_n(t)$ are the time-dependent thermal and structural generalized coordinates.

Assuming that the material properties (elasticity tensor, coefficient of thermal expansion) do not vary with temperature, it is found [3] that (summation over repeated indices assumed)

$$M_{ij}\ddot{q}_j + D_{ij}\dot{q}_j + \left[K_{ij}^{(1)} - K_{ijl}^{(th)}\,\tau_l\right] q_j + K_{ijl}^{(2)} q_j q_l + K_{ijlp}^{(3)} q_j q_l q_p = F_i + F_{il}^{(th)}\,\tau_l \quad . \quad (3)$$

In this equation, M_{ij} denotes the elements of the mass matrix, $K_{ij}^{(1)}$, $K_{ijl}^{(2)}$, $K_{ijlp}^{(3)}$ are linear, quadratic, and cubic stiffness coefficients and F_i are the modal mechanical forces. The parameters $K_{ijl}^{(th)}$ and $F_{il}^{(th)}$ represent the sole coupling terms with the temperature field and can be expressed, see [3], as the discretization of

$$K_{mnp}^{(th)} = \int_{\Omega_0} \frac{\partial U_i^{(m)}}{\partial X_k} \frac{\partial U_i^{(n)}}{\partial X_j} C_{jklr}\,\alpha_{lr}\, T^{(p)}\, d\underline{X} \; ; \; F_{mn}^{(th)} = \int_{\Omega_0} \frac{\partial U_i^{(m)}}{\partial X_k} C_{iklr}\,\alpha_{lr}\, T^{(n)}\, d\underline{X} \quad (4),(5)$$

In these equations, Ω_0 denotes the domain of the structure in the undeformed configuration, $U_i^{(m)}(\underline{X})$ is *i*th component (*i* =1, 2, 3) of the *m*th basis function for the representation of the continuous displacement field, C_{iklr} and α_{lr} are components of the 4th order elasticity tensor and thermal expansion matrix respectively.

The equations for the heat conduction are [3]

$$B_{ij}\ \frac{d\left(\tau_j\right)}{dt}+\tilde{K}_{ij}\tau_j=P_i \tag{6}$$

where B_{ij} and $\tilde{K}_{ij}$ are the capacitance and conductance matrices of the finite element model, assumed independent of temperature. The source term P_i represents the combined effects of an applied flux, nonzero homogenous boundary conditions, radiation, latency, etc.

Uncertainty will be assumed here to affect the capacitance and conductance matrices as well as the thermal coupling terms in the structural equations. These 3 groups of terms will be assumed to be stochastically independent of each other and will be modeled within the maximum entropy framework under the assumptions that (i) B_{ij} and $\tilde{K}_{ij}$ are symmetric and positive definite matrices, (ii) $K_{ijl}^{(th)}$ is a symmetric matrix for any value of *l*, and (iii) $K_{ijl}^{(th)}$ and $F_{il}^{(th)}$ involve the same kernel describing the thermal expansion of the structure.

The above methodology will be demonstrated on a representative hypersonic vehicle panel under a “one-way coupled” constant rate ascent trajectory analysis [4] with the mean structural and thermal reduced order models developed in [5,6].

REFERENCES

[1] C. Soize, A nonparametric model of random uncertainties for reduced matrix models in structural dynamics. *Probabilistic Engineering Mechanics*, Vol. **15**, pp. 277-294, 2000.

[2] C. Soize, A comprehensive overview of a non-parametric probabilistic approach of model uncertainties for predictive models in structural dynamics. *Journal of Sound and Vibration*, Vol. **288**, pp. 623-652, 2005.

[3] R. Perez, X.Q. Wang and M.P. Mignolet, Nonlinear reduced order models for thermoelastodynamic response of isotropic and FGM panels. *AIAA Journal*, Vol. **49**, pp. 630-641, 2011.

[4] A. Culler and J.J. McNamara, Impact of fluid-thermal-structural coupling on response prediction of hypersonic skin panels, *AIAA Journal*, Vol. **49**, pp 2393-2406, 2011.

[5] A.K. Matney, R.A. Perez, S.M. Spottswood, X.Q. Wang, and M.P. Mignolet, Nonlinear structural-thermal reduced order modeling of a representative hypersonic structure. *Proceedings of the 53nd Structures, Structural Dynamics and Materials Conference,* Honolulu, Hawaii, Apr. 23-26, AIAA Paper AIAA-2012-1972, 2012.

[6] A.K. Matney, S.M. Spottswood, M.P. Mignolet, A.J. Culler and J.J. McNamara, Coupled reduced order model-based structural-thermal prediction of hypersonic panel response. *Proceedings of the 11th International Conference on Recent Advances in Structural Dynamics,* Pisa, Italy, Jul. 1-3, 2013.