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ORTHOTROPIC SIMO AND PISTER HYPERELASTICITY THEORY

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In this work we have developed the first hyperelastic strain energy function for orthotropic continua that is able to map the same logical properties of advanced isotopic hyperelastic constitutive laws. In particular, we choose the model of Simo and Pister (1984) and physically replicate the model in orthotropy by use of Intrinsic-Field Tensors. First, we show that the model can be represented by a standard archetypal equation for strain energy. We expand this equation out to an uncompressed form of quadruple contractions between fourth-order tensors, rather than of scalar products of scalar invariants. In the final step, the Lamé parameters of Simo and Pister's model are replaced by a proposed orthotropic form – scalars replaced by fourth-order tensors – and then interchange the classical strain tensors with advanced Intrinsic-Field Tensors of the equivalent order of strain measure. The resulting model collapses back down to the isotropic form by nothing more than equality of parameters in all directions (isotropy). We propose that the new model is not an orthotropic 'equivalent', but actually the parent form of the model, which essentially represents Simo and Pister hyperelasticity without the luxury of isotropic material properties, without the ease of representation by scalar invariant and without the simplicity afforded by classical symmetric strain and deformation tensors. The orthotropic hyperelastic theory presented here represents the archetype for a comprehensive new elasticity theory called Orthotropic Continuum Mechanics.

Hyperelastic materials are a class of solids that can be modelled as continua with rate-independent strain energy defined purely as a function of deformation and the material parameters. The Simo and Pister model [1], like most hyperelastic Strain Energy Functions (SEFs), is restricted to isotropic materials; one of its particular benefits is that pure distortional deformation is independent of the volumetric modulus for finite strain, and that the volumetric strain is a logarithmic function of deformation. From a mathematical standpoint, the scalar strain energy function is expressed as the product of scalar deformation invariants and scalar coefficients.

In this paper we posit that there is a generalised form of the SEF that a large class of hyperelastic functions should be able to be written within, and that those that cannot *can* either be closely approximated by the general form or do not satisfy certain expected boundary conditions of finite strain hyperelasticity. Correspondingly, use of the model guarantees the desired properties of many widely used isotropic models irrespective of its implementation. This general form of SEF is an abstraction one level up of the classical SEF and is mathematically encompassing due to its field variables that allow it to represent many difference SEFs in the same way that Seth–Hill strain can encompass most strain measures; we called it the Generalised Strain Energy (GSE).

After first demonstrating that there is an exact representation for the isotropic Simo and Pister strain energy within the GSE, we further revisit the class of orthotropic tensors that are asymmetric and of the form of Intrinsic-Field Tensors (IFTs) [2]. We also propose a natural separation of the extended form of the Hookean material tensor for stiffness, which is naturally extended for IFTs such that it utilises all free terms within a fourth order tensor having major symmetry. These are the orthotropic Lamé material tensors for stiffness and compliance.

These tools allow a new model for orthotropic Simo and Pister hyperelasticity that we purport to be the first of its kind and the only such model to inherit and maintain so many logical properties of isotropic hyperelasticity, structural tensors [3] and orthotropic material models simultaneously. Since the proposed model achieves these features by derivation and as pure theoretical development, the properties are ensured.

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