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ROUTE TO CHAOS IN MINIMAL PLANE COUETTE FLOW

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Transition to turbulence in wall-bounded flow such as pipe flow or plane Couette flow is essentially caused by finite amplitude disturbances, and has been one of the biggest open problem in fluid dynamics. Recent works by Kreilos & Eckhardt[1] and Avila et al.[2] showed numerically the onset of chaos in plane Couette flow and pipe flow respectively. In both works, besides periodicity, they imposed additional symmetry conditions and successfully obtained the stable nonlinear solutions, which are the upper-branches arising from saddle-node bifurcations to be the origin of chaotic attractors via period-doubling cascade or torus explosion. They also showed that the chaotic attractors lose their stability within narrow ranges of Reynolds number (Re).

Here we consider incompressible Newtonian flow in a minimal plane Couette system with the domain size $1.755\pi \times 2 \times 1.2\pi$. (see for example, Kawahara[3]) No additional symmetry is imposed besides the periodic boundary conditions. We integrate numerically this system using spectral-Galerkin method and investigate final flow state for $236 \leq Re \leq 247$.

Figure 1 displays the bifurcation of this systems as the value of the local maximum E_{cf} of the cross-flow energy[1] as a function of Re. Laminar flow(LF), which corresponds to $E_{\mathrm{cf}} = 0$, is linearly stable. For $236.1 \leq Re \leq 246.6$, several attractors coexist with LF. The onset of the nonlinear solutions is the pair of periodic orbits (P2) caused by saddle-node bifurcation [3], and each of them has two local maxima per cycle but these have the same value. (This is also the case for P6.) The upper-branch of P2 ($\mathrm{UB_{P2}}$) loses stability at $Re = 246.1$[3] through a supercritical Neimark-Sacker bifurcation resulting in the stable torus. P4 and P6 also appear from saddle-node bifurcation and each of them leads to a chaotic attractor via period-doubling.

Important global bifurcations are discussed below. At $Re_{c1} = 240.4$ the boundary crisis occurs between $\mathrm{UB_{P2}}$ and the chaotic attractor, and the chaotic trajectory can approach to any neighborhood of the $\mathrm{LB_{P4}}$. Above this Re the chaotic attractor is replaced by the chaotic saddle and all trajectories in its neighborhood are attracted to $\mathrm{UB}_{\mathrm{P2}}$. This chaotic set also touches $\mathrm{LB}_{\mathrm{P2}}$ at $Re_{c2} = 240.9$ leading to to a homoclinic tangle[4] on the edge of LF basin, and the fractal basin boundaries appear between $\mathrm{UB}_{\mathrm{P2}}$ and LF (Figure 2). Above Re_{c3} the chaotic attractor originating from P6 disappear and trajectories starting in its neighborhood decay to LF. At Re_{c3} the chaotic orbit does not touch any LB in Figure 1. Finally at Re_{c4} the collapse of the torus occurs and there is no attractor except for LF. At the conference, we will discuss more detail about the formation/destruction of the fractal basin boundaries caused by the boundary crises and the heteroclinic connections among the periodic orbits.

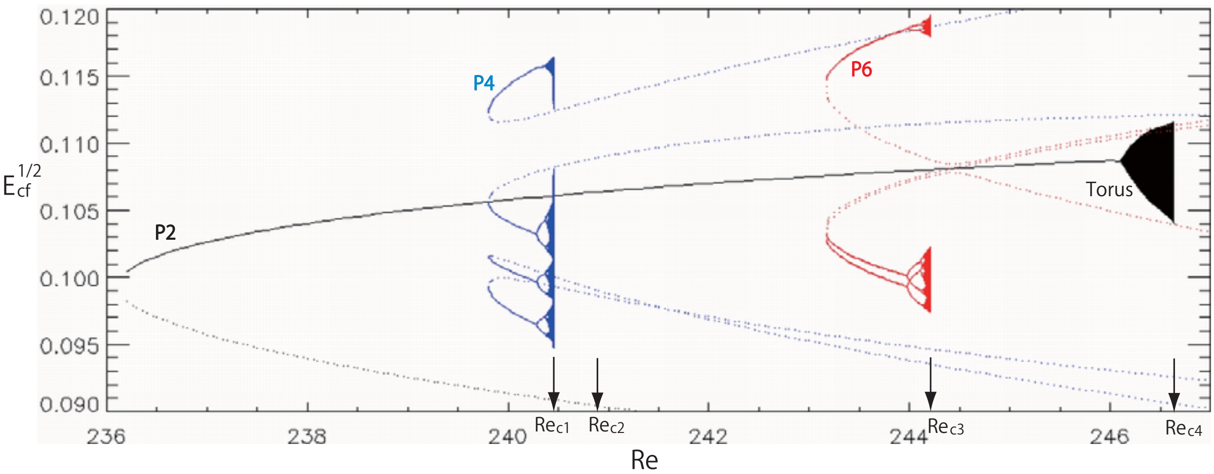

Figure 1: Bifurcation diagram of minimal plane Couette flow for $236 \leq Re \leq 247$. Lines and filled areas represent attractors and dotted lines are saddles. At $Re = 236.1, 239.8$ and 243.2 periodic orbits are created by saddle-node bifurcation. During one cycle these have two, four and six local maxima of E_{cf} respectively and have stable upper branch (P2, P4 and P6). Major global bifurcations are found at $Re_c = 240.4, 240.9, 244.2$ and 246.6 , which are indicated by arrows

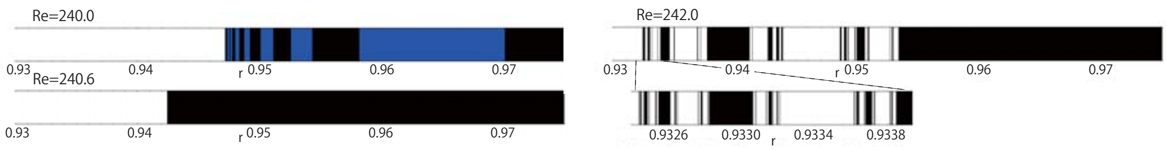

Figure 2: Section of attraction basins of laminar flow(white), P2(black) and P4(blue) along a line in phase space. Variable r on the horizontal axis denotes the distance from the origin (laminar flow).

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