

Universal Meshes: High-Order Simulation of Problems with Evolving Geometries

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Multiple problems in engineering involve geometries that evolve with the problem. Fluid-structure interaction, phase transformation, and shape optimization problems are the most common, but crack propagation problems and solids undergoing extreme deformations need such strategies as well.

Three types of approaches are typically adopted for these problems: periodic remeshing, arbitrary Lagrangian-Eulerian kinematic descriptions, and embedded or immersed boundary methods. The first one is generally considered computationally expensive, the second one breaks down under very large deformations, and the last one often leads to low-order methods because of a poor representation of the geometry.

In this talk, I will introduce the concept of "Universal Meshes", which combines the best of each one of the above strategies. In a nutshell, a Universal Mesh for a class of domains is a triangulation that is able to mesh any of the domains in the class upon minor perturbations of the positions of its nodes. Hence, as the domain evolves, the perturbed universal mesh provides an exact triangulation of the geometry. It is then possible to formulate high-order methods for problems with evolving geometries in a standard way.

I will show the application of these ideas to hydraulic fracturing and ballistic penetration problems. In the former, in which a crack in an elastic medium advances due to a pressurized fluid in its interior, the universal mesh is used to exactly mesh the faces of the evolving crack. This enables the coupled solution of the elasticity equations in the domain, and the lubrication equations for the motion of the fluid on the crack manifold. In contrast, for ballistic penetration problems, these ideas are used to periodically remesh the domain of the deforming solid. Along the way, I will briefly highlight other ideas related to time-integration methods, which we created for these two problems as well.

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