11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A FORCE-BASED FORMULATION FOR THE ANALYSIS OF FRAMES WITH NON-HOLONOMIC HARDENING PLASTIC HINGES

Theodoros N. Patsios[1]* and Konstantinos V. Spiliopoulos[2]

[1] PhD Candidate, ISAAR, School of Civil Engineering, NTUA, Heroon Polytechniou 9, Zografou, 15780, Athens, Greece, tpatsios@central.ntua.gr, http://users.ntua.gr/tpatsios/
[2] Assoc. Professor, ISAAR, School of Civil Engineering, NTUA, Heroon Polytechniou 9, Zografou, 15780, Athens, Greece, kvspilio@central.ntua.gr, http://users.ntua.gr/kvspilio/

Key Words: *Numerical Methods, Force Method, Non-Holonomic Plasticity, Plastic Hinges, Material Hardening, Non-Linear Analysis, Framed Structures, Planar Frames, Mathematical Programming, Quadratic Programming.*

INTRODUCTION

The study of post-elastic behaviour of framed structures has gained significant interest in the past years, mainly due to the need for a proper estimation of the bearing capacity of old structures with respect to the continuously increasing earthquake resistance demands, and the need for achievement of more economic design for new as well as for retrofitted structures. In engineering practice, the dominant approach to nonlinear analysis with plastic hinges of zero length is the direct stiffness method, mainly because it is considered as easier to automate.

However, Maier [1] has proved that mathematical programming offers an ideal framework for non-holonomic plasticity; material hardening/softening is also included, via associated plastic flow rules ([2]−[4]). Furthermore, using redundant forces as the basic (*primary*) variables is a better choice than nodal displacements, mainly due to the smaller number of unknowns [5]; efficient and simple to implement techniques for automating the force method have been already presented in the literature [6].

Herein, an existing efficient force-based method for step by step non-linear analysis [7] is extended to include material hardening; a brief description follows below.

MAIN FEATURES

Associated plastic flow is added to an existing formulation [7]. Non-linear behaviour of the structure's material is approximated by piece-wise linear (PWL) constitutive laws ([2]−[4]). Compatibility, static admissibility, and linear complementarity between plastic potential and non-associated plastic deformations, serve as the Karush−Kuhn−Tucker (KKT) conditions of the following optimization problem:

$$Minimize: \ f(\Delta\mathbf{p}_k) = \frac{1}{2}\cdot\Delta\mathbf{p}_k^T\cdot[\bar{\mathbf{B}}^T\cdot(\bar{\mathbf{F}}+\bar{\mathbf{S}})\cdot\bar{\mathbf{B}}]\cdot\Delta\mathbf{p}_k + \Delta\gamma_k\cdot\Delta\mathbf{p}_k^T\cdot[\bar{\mathbf{B}}^T\cdot(\bar{\mathbf{F}}+\bar{\mathbf{S}})\cdot\bar{\mathbf{B}}_0]\cdot\mathbf{r}_p$$

$$Subject\ to: \ (\bar{\mathbf{N}}^T\cdot\bar{\mathbf{B}})\cdot\Delta\mathbf{p}_k \le \mathbf{c}_k - \bar{\mathbf{N}}^T\cdot\mathbf{Q}_{k-1} - \Delta\gamma_k\cdot(\bar{\mathbf{N}}^T\cdot\bar{\mathbf{B}}_0)\cdot\mathbf{r}_p$$

Wherein the formulation above, matrix "$\bar{\mathbf{S}}$" expresses each critical section's associated plastic flow rate in terms of flexibility and constant's vector "$\mathbf{c}_k$" expresses −in unit-free quantities− the limits of each critical section's yield locus. Both "$\bar{\mathbf{S}}$" and "$\mathbf{c}_k$" are updated w.r.t. the material's piece-wise linear (PWL) constitutive law. All other matrices are explained in [7].

Prior to every incremental step "*k*", for all critical sections whose stress configuration is within the elastic region or follows a perfectly plastic flow, elastic-perfectly plastic behaviour is assumed ($\bar{\mathbf{S}} = \mathbf{0}$), while for all critical sections whose stress configuration lies along an associated plastic flow branch of the PWL curve, holonomic plastic behaviour is assumed ($\bar{\mathbf{S}} \neq \mathbf{0}$). For every step "$k$", a "trial and error" sub-step pattern is followed: Each time, beginning from a well-defined stress/deformation state, a "trial" sub-step that satisfies the linear complementarity condition of non-holonomic plasticity is run; based on any constraints that may develop Lagrange multipliers (*this points to the critical sections that leave the elastic region to enter a plastic flow phase*) and/or any stress increments that may violate the irreversibility of their associated plastic flow (*this points to the critical sections that return to the elastic region from an associated plastic flow phase*), the required parts of matrix "$\bar{\mathbf{S}}$" and of matrix "$\mathbf{c}_k$" are updated so that they signal initiation of associated plastic flow and predefine the direction and limits of its evolution (*while they define an admissible continuous domain for the stress increments*), or termination (*plastic unstressing – non-holonomic plasticity*). Then, an "error" sub-step is run, that begins from the stress/deformation state where the "trial" sub-step began from, but uses the previously modified matrices; this "error" sub-step finds the correct feasible direction. Any suitable algorithm may be used, e.g. [8].

CONCLUSIONS

The proposed formulation renders robust, and faster than the equivalent direct stiffness method; examples and comparisons with widely accepted commercial packages will be presented. The method may be extended to 2nd order analysis [9].

ACKNOWLEDGMENTS

The work presented herein is part of T.N. Patsios' PhD Thesis, conducted under the advisory of Assoc. Professor K.V. Spiliopoulos, and partially funded by NTUA's Special Account for Research Grants (ΕΛΚΕ). The 1st author was awarded with an IACM Scholarship for his participation in the WCCM-ECCM-ECFD 2014. All support is acknowledged.

REFERENCES

[1] Maier, G.. A quadratic programming approach for certain classes of nonlinear structural problems. Meccanica 3: Issue 2, pp.121–130, 1968.

[2] Maier, G.. A Matrix Structural Theory of Piece Wise Linear Elastoplasticity with Interacting Yield Planes. Meccanica, pp.54–66, 1970.

[3] Maier, G.. On Elastic-Plastic Structures with Associated Stress-Strain Relations Allowing for Work Softening. Meccanica, pp.55–64, 1967.

[4] Cocchetti, G., Maier, G.. Elastic–plastic and limit–state analyses of frames with softening plastic–hinge models by mathematical programming. Int. J. Solids & Struct. 40, pp.7219–7244, 2003.

[5] Spillers, W.R.. Application of topology in structural analysis. J. Struct. Div. ASCE 89, pp. 301-313, 1963.

[6] Spiliopoulos, K.V.. On the automation of the force method in the optimal plastic design of frames. Comput. Methods Appl. Mech. Engrg 141: Issues 1–2, pp.141–156, 1997.

[7] Spiliopoulos, K.V., Patsios, T.N.. An Efficient Mathematical Programming Method for the Elastoplastic Analysis of Frames. J. Eng. Struct. 32: Issue 5, pp.1199–1214, 2010.

[8] Goldfarb D, Idnani A. A numerically stable dual method for solving strictly convex quadratic programs. Math Program 27: pp.1–33, 1983.

[9] Patsios, T.N., Spiliopoulos, K.V.. A force-based formulation for the 2nd order elastoplastic analysis of frames. *Proc. 2nd ECCOMAS YIC*, pp.104–108; Bordeaux, France, 2–6 Sep 2013.