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NUMERICALLY ROBUST PARALLEL SWEEPING PRECONDITIONERS

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Moving Perfectly Matched Layers sweeping preconditioners [2] have been shown to yield a quasi-direct, $O(N^{4/3})$ setup and $O(N \log N)$ solve, solution scheme for heterogeneous time-harmonic wave equations without large-scale resonances [6, 3, 7]. One of the challenges in extending these methods to distributed-memory architectures is that each *sweep* involved in applying the preconditioner requires solving many small sparse-direct systems one after another. It was shown in [3] that a careful usage of *selective inversion* [5] can overcome the poor scalability of traditional sparse-direct triangular solves and yield a scalable quasi-direct solver for certain heterogeneous Helmholtz equations.

One of the shortcomings of the previous work is that it did not allow for the use of pivoting, which is well-known to be generally necessary for the accurate solution of indefinite systems. We have therefore implemented an extension of selective inversion from unpivoted LDL factorizations to Bunch-Kaufman pivoting [1] restricted to diagonal blocks. We also show that, when a significant number of linear systems are to be solved at once, that distributing the right-hand sides in a 2D manner allows for sparse-direct triangular solves to approach the parallel performance of selected inversion. That is, numerically troublesome direct inversion can be avoided.

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