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A PARTITION OF UNITY METHOD FOR A CLASS OF FOURTH ORDER ELLIPTIC VARIATIONAL INEQUALITIES

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Elliptic fourth order variational inequalities are important problems in computational mechanics. As examples we will consider the displacement obstacle problem for simply supported Kirchhoff plates and an elliptic distributed optimal control problem with pointwise state constraints. The conforming finite element method (FEM) is expensive to use for these problems due to the necessity of C^1 elements.

The convergence analysis of these problems is also cumbersome due to the lack of full elliptic regularity. In fact, it is known [1, 3] that the solution to these problems is only in $H^{2+\alpha}(\Omega)$, for $\alpha \in (0,1)$, even for smooth data. Following techniques presented in [2], one can obtain the convergence rate of $\mathcal{O}(h^\alpha)$ in the energy norm for conforming, nonconforming, and discontinuous Galerkin methods for this problem.

In this talk we present an enriched partition of unity method for these problems. With the appropriate choice of enrichment functions, we show that this method is able to obtain the optimal rate of convergence $\mathcal{O}(h)$ in the energy norm. This talk will consist of details of the method, convergence analysis, and numerical examples.

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