11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

STABILISATION OF DISCRETE ADJOINT SOLVERS THROUGH IMPROVED PRIMAL TIMESTEPPING

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Key words: *Discrete adjoint, automatic differentiation, GMRES.*

One major challenge in applying the adjoint method to large industrial cases is the lack of robustness of the flow and adjoint solvers. Discrete adjoints transpose the Jacobian of the system of discretised equations exactly, and hence inherit the linear stability from the primal flow solver. Typically the linear stability properties of the iterative scheme of the flow solver is very well understood, hence this is an attractive property. When the flow solver does not converge fully due to either numerical or physical instabilities, the discrete adjoint is bound to diverge and force the gradient-based optimisation to stop prematurely.

The generalised minimal residual method (GMRES) and the recursive projection method (RPM) have been proposed to stabilise the solver in the presence of solver instability [1, 2], and proved to be effective for cases of moderate size [3]. The main issue with either GMRES and RPM is inconsistency in that the strong linear or adjoint solvers equipped with GMRES or RPM are applied to a flow solution that is only partially converged to low residual level due to solver instability, and thus although the adjoint solver will converge fully, the adjoint solution will depend on the non-unique flow solution [4]. The second drawback of using either GMRES or RPM is that the memory requirement due to using more search directions by storing more snapshots will be highly case independent and typically result in prohibitive memory overhead for large cases [5].

To circumvent the weakness of the existing methods for stabilising the adjoint solver, a novel method called Jacobian-Trained Krylov Implicit-Runge-Kutta (JT-KIRK) is proposed in this work.

The novel JT-KIRK algorithm uses the 1st-order approximate Jacobian matrix which is available already through automatic differentiation when building the adjoint solver. The Jacobian is used to construct an efficient preconditioner for a restarted GMRES solver. To make the GMRES solver a suitable smoother for multigrid, it is used in a multi-stage

Runge-Kutta time marching scheme. The JT-KIRK algorithm is used to iterate the flow solver and is also used to iterate the adjoint solver in the same way. The resulting suite of solvers are thus consistent in that the final converged adjoint solution is based on the unique fully converged flow solution. In addition, due to the superior smoothing property of the RK scheme on the high frequency error modes, multigrid can be effectively used to further stabilise and accelerate the solver convergence.

The JT-KIRK solver is applied to industrial test cases to demonstrate the solver robustness enhancement for unstable cases and the convergence acceleration for stable cases. The results are further examined using eigen-analysis to identify the most problematic eigenmodes, proving with firm evidence that the solver instability can rise from either numerical or physical instability of the flows.

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