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RELAXING THE CFL NUMBER OF THE DISCONTINUOUS GALERKIN METHOD

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The Courant-Friedrichs-Lewy (CFL) number of the discontinuous Galerkin method (DGM) applied to hyperbolic conservation laws decreases with the order of approximation p. We explain the reasons for this order-dependent stiffness and propose a way to relax it without reducing the formal order of accuracy of the method. We apply a von Neumann type analysis to the DGM and prove that the amplification factor of the scheme is given by a rational function that is the subdiagonal Pade appoximant of the exponent. This approximant is then related to the eigenvalues of the discretization matrix, dispersion and dissipation errors (i.e. the superaccuracy) and the spatial superconvergence. Modifying the rational approximant leads to a family of methods that might have smaller spectra, i.e. larger CFL numbers, while retaining the same formal convergence rate. However, the modification results in larger dispersion/dissipation errors as these two are given in terms of the same rational function. We discuss the limit to which the CFL can be improved. In particular, we argue that the stiffness of the DGM is due not to the properties of high-order polynomials in general but to the specific polynomials lying at the base of the method. Finally, we discuss the trade-offs between accuracy and the CFL restriction and remark when choosing a scheme with a larger CFL might be beneficial.

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