11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

ON THE USE OF HIGH-ORDER STATISTICS IN ROBUST DESIGN OPTIMIZATION

P.M. Congedo[1] and G. Geraci[2] and G. Iaccarino[2]

[1] INRIA Bordeaux Sud-Ouest, Team Bacchus, 200 avenue de la Vieille Tour, 33405 Talence, France. pietro.congedo@inria.fr, gianluca.geraci@inria.fr.
[2] Mechanical Engineering Dept, Center for Turbulence Research, Stanford University, Bld 500, CA 94305-3035, USA. jops@stanford.edu.

Key words: *High-order statistics, Robust design optimization, CFD.*

Optimization and design in the presence of uncertain operating conditions, material properties and manufacturing tolerances poses a tremendous challenge to the scientific computing community. In many industry-relevant situations the performance metrics depend in a complex, non-linear fashion on those factors and the construction of an accurate representation of this relationship is difficult. Probabilistic uncertainty quantification (UQ) approaches represent the inputs as random variables and seek to construct a statistical characterization of few quantities of interest. Several methodologies are proposed to tackle this problem; polynomial chaos (PC) methods [1] can provide considerable speed-up in computational time when compared to MC. In realistic situations however, the presence of a large number of uncertain inputs leads to an exponential increase of the cost thus making these methodologies unfeasible [2]. This situation becomes even more challenging when robust design optimization is tackled [3]. Robust optimization processes may require a prohibitive computational cost when dealing with a large number of uncertainties and a highly non-linear fitness function. Efforts in the development of numerical method are directed mainly to reduce the number of deterministic evaluations necessary for solving the optimization problem and for the uncertainty quantification (UQ) of the performances of interest. The overall cost is typically the product of the cost of the two approaches because the stochastic analysis and the optimization strategy are completely decoupled. Decoupled approaches are simple but more expensive than necessary.

Several UQ methods have been developed with the objective of reducing the number of solution required to obtain a statistical characterization of the quantity of interest. An alternative solution is based on approaches attempting to identify the relative importance of the input uncertainties on the output. A well known methodology is based on a decomposition of the variance of the quantity of interest in contributions closely connected to each of the input uncertainties (main effects) or combined inputs [5]. Recently, a practical way to decompose high-order statistical moments has been also proposed [4].

In this work, we illustrate the impact of high-order statistics in robust design efficiency and global computational cost. The aim is to provide some useful indications for obtaining a good trade-off between the high-quality information given by high-order statistics and the feasibility of the whole optimization loop. Several test-case will be presented and analyzed. Moreover, an efficient multi-objective optimization method taking into account high-order statistic moments, such as the third and fourth-order statistic moments, i.e. skewness and kurtosis, respectively, will be proposed. These moments can be easily computed by means of a Polynomial Chaos (PC) method. The efficiency of the method in terms of computational cost and fitness function complexity will be assessed in a realistic CFD-case optimization problem.

REFERENCES

[1] Xiu, Dongbin and Karniadakis, George Em. The Wiener–Askey Polynomial Chaos for Stochastic Differential Equations. *SIAM Journal on Scientific Computing*, Vol. **24**, 619–644, 2002.

[2] Foo, Jasmine and Karniadakis, George Em. Multi-element probabilistic collocation method in high dimensions. *Journal of Computational Physics*, Vol. **229**, 1536–1557, 2010.

[3] M. S. Eldred, C. G. Webster, P. G. Constantine. Design Under Uncertainty Employing Stochastic Expansion Methods. *AIAA Paper 20086001*, 2008.

[4] Abgrall, R. and Congedo, P.M. and Geraci, G. and Iaccarino, G.. Decomposition of high-order statistics. *INRIA Research Report*, **RR-8193**, 2012.

[5] Congedo, P.M. and Geraci, G. and Abgrall, R. and Pediroda, V. and Parussini, L.. TSI metamodels-based multi-objective robust optimization. *Engineering Computations*, Vol. **30-8**, 1032–1053, 2013.