

## GRADIENT SMOOTHING FOR NEARLY INCOMPRESSIBLE HYPERELASTICITY

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We present a strain smoothing approach for nearly incompressible finite elasticity, which we test on neo-Hookean materials. The aim of strain smoothing approaches, the smoothed finite element method (SFEM) in particular, is to improve the quality of simplex elements. Since non-structured simplex meshes of complex volumes are much more easily generated than structured meshes of hexahedral elements, improving the quality of simplex elements could reduce the human intervention required in the simulation of solids of complex shapes. We are particularly interested in human organs, for which generating hexahedral dominant meshes can be extremely challenging.

There are several possibilities to achieve such a goal, for example: mixed formulations, enhanced assumed strain, reduced integration, and the strain smoothing method. Strain smoothing relies on stabilized conforming nodal integration (SCNI) [3], which Liu et al. [4] applied within a finite element context ([2] presents a review, [5]). The main features of SFEM are: 1) insensitivity to mesh distortion because of absence of isoparametric mapping, 2) does not require the derivatives of the shape functions, 3) in selected cases, lower computational cost than FEM at the same accuracy level, 4) any  $n$ -sided/faced polygonal/edra elements can be built, 5) alleviate volumetric locking. Despite these positive points, the method was not used, to the authors' knowledge in the context of non-linear elasticity.

In this paper, we investigate numerically the properties of edge-based strain smoothing for 2D finite and quasi incompressible elasticity and show that the method is able to

successfully alleviate volumetric locking.

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