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LOW ORDER OR HIGH ORDER; THIS IS THE QUESTION!

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Many efficient algorithms that were initially developed for structured grids, such as Yee's scheme for the solution of the Maxwell equations and the MAC algorithm for the solution of the Navier-Stokes equations can be regarded as co-volume solution techniques. Traditional co-volume methods, which are implemented on a pair of mutually orthogonal cartesian meshes, have been shown to exhibit a high degree of computationally efficiency, in terms of both CPU and memory requirements compared to, for example, a finite element method. In the unstructured mesh environment, a Delaunay triangulation and the corresponding Voronoi diagram provide a natural pair of meshes. In 2D, corresponding edges of the Voronoi and Delaunay meshes are mutually orthogonal. In 3D, every edge of the Voronoi diagram is orthogonal to the corresponding face of the Delaunay triangulation and vice versa. Co-volume discretisations can be devised on such unstructured meshes, so that, theoretically, it should be possible to use them for the solution of problems involving industrially complex geometries.

On the other hand, during the last decade, there has been a great interest in developing efficient high-order methods for computational fluid dynamics and computational electromagnetics. An example is the EU supported ADIGMA project, which was aimed at developing innovative adaptive high-order methods for the next generation of industrial aerodynamic flow solvers. The results of this project indicated that high-order methods are able to reduce drastically the required number of degrees of freedom but, normally, at the price of increasing the computational cost.

The talk will highlight the methods used to generate the required mesh quality that is necessary for algorithms that are based on co-volume methods and for algorithms using a high order finite element method. First, a review of the effectiveness of optimisation procedures, employing both gradient free and gradient based techniques, to generate the two mutually orthogonal meshes appropriate for the use with the co-volume methods will be given [1]. Second, we will present a practical method that is based on a linear elastic model to eliminate the non-valid elements that may result from curving a linear mesh during the process of generating high order meshes [2]. Particular attention is paid to the generation of anisotropic boundary layer meshes, suitable for the efficient simulation of high Reynolds number fluid flow problems. This is a crucial problem that must be overcome, before high-order methods can be routinely applied by the computational fluid dynamics community to the solution of problems of industrial interest. Finally, we will attempt to identify the class of problems for which high order methods will have advantages over traditional low order methods on unstructured grids. Practical test cases, which utilise the generated meshes using the extended MAC algorithm, the modified Yee algorithm and the high order finite element method, will be used for this purpose [3, 4].

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