

AN EULERIAN-LAGRANGIAN WENO SCHEME FOR NONLINEAR CONSERVATION LAWS

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Key words: *Hyperbolic transport, Semi-Lagrangian, Finite volume, Locally conservative, Characteristics, Traceline, Re-average.*

We develop a formally high order Eulerian-Lagrangian WENO finite volume scheme for nonlinear scalar conservation laws that combines ideas of Lagrangian traceline methods with WENO reconstructions. The particles within a grid element are transported in the manner of a standard Eulerian-Lagrangian (or semi-Lagrangian) scheme using a fixed velocity v . A flux correction computation accounts for particles that cross the v -traceline during the time step. If $v = 0$, the scheme reduces to an almost standard WENO5 scheme. The CFL condition is relaxed when v is chosen to approximate either the characteristic or particle velocity. Excellent numerical results are obtained using relatively long time steps.

The v -traceback points can fall arbitrarily within the computational grid, and linear WENO weights may not exist for the point. A general WENO technique is described to reconstruct to any order the integral of a smooth function using averages defined over a general, nonuniform computational grid. Moreover, to high accuracy, local averages can also be reconstructed. By re-averaging the function to a uniform reconstruction grid that includes a point of interest, one can apply a standard WENO reconstruction to obtain a high order point value of the function.