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A MESHLESS APPROACH BASED ON THE CELL METHOD FOR BONE BIOMECHANICS

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Subject-specific numerical models based on medical imaging can constitute a powerful tool to predict mechanical behaviour of bony segments. Usually the Finite Element Method (FEM) is used. Both the geometry and the mechanical properties are usually derived from calibrated Computed Tomography (CT) which provides a scalar field related to the local density over a regular lattice. An empirical density-elasticity relationship is used to associate the mechanical properties to each element, depending on its positioning over the CT lattice. A direct conversion of the CT voxels into the hexahedral brick elements is possible, but it produces jagged surfaces and related mechanical artefacts. Alternatively, an unstructured mesh can be produced once the precise definition of the bone surface is provided. The mesh definition still remains a semi-automatic process which results time expensive and labour intensive and the need of expert skills constitutes a lack of automation which might limit clinical applications.

Meshless approaches may overcome the difficulties in defining a good mesh. Nevertheless, they seem to have had a limited application in biomechanics and were adopted mainly for soft tissues. So far, only few studies reported the adoption of a meshless method for bone mechanics and no validation was provided. This is probably due to the theoretical complexity and to the difficulties in imposing boundary conditions, probably the main drawback of many meshless approaches based on the differential formulation. Recently, based on the Cell Method [1], some meshless approaches were proposed [2], which are not affected by complexities in imposing boundary conditions. The Cell Method (CM) is a numerical method based on direct algebraical formulation of physical laws. It uses global variables associated to the geometrical elements of two mesh complexes: the primal cell complex (similar to a traditional FEM mesh) offers the reference for the configuration variables; the dual cell complex, which is obtained dividing each primal cell in parts to be associated to each primal vertex, collocates the source variables. The dual cell constitutes the tributary region used to write the balance equations. Regarding elasticity, the unknown field (displacements at each primal vertex) is interpolated by a polynomial. In each primal cell: (i) the strain tensor is calculated in terms of nodal displacements; (ii) the constitutive law expresses the relationship between stress and strain tensor; (iii) the

Cauchy's relationship expresses the force acting on an oriented flat surface. Combining these equations, the force acting on a flat surface contained in a given primal cell can be expressed in term of its nodal displacements. The solving algebraical system is obtained assembling the equilibrium equation at each dual cell. This equilibrium equals the volume forces B_h collected by the dual cell h to the surface forces T_h^c acting on each part of its boundary (which belong to different primal cells c):

$$\sum T_h^c = B^c \tag{1}$$

It is not necessary to know the whole primal complex for writing each balance equation. Furthermore, the balance equation does not depend on the shape of the tributary region. This fact leads to a straightforward meshless approach: the primal cell complex can be locally defined connecting each primal node with a number of neighbouring nodes. The dual cell surrounding the studied node can then be defined and the balance equation can express the relationship between the displacements of the local cloud of nodes and the forces collected by the dual cell.

Local primal cells permit the local interpolation of the unknown field. On the other hand, the tributary region adopted for the balance equation is arbitrary. A so-called "truly meshless" approach is easily achievable: assuming a local cloud of primal nodes surrounding a given node, an elementary tributary region (square, cube, circle, sphere) can be adopted and the unknown field can be locally interpolated by a polynomial of a proper degree or a radial basis function interpolation. The analytical expression of the surface forces is derived by the function which defines the strains as the symmetric part of the displacement gradient via the constitutive law. Since it constitutes a collocation approach, no problems arises in imposing the essential boundary conditions. On the other hand, distributed or concentrated forces are always collected by the tributary regions locally defined around each primal nodes.

In this work, the formulation of the above mentioned meshless approaches based on the CM is presented with some verification tests for the elastostatics. The application of the local mesh approach to the prediction of strains over bone segments surfaces is also presented and results of a validation study (involving 8 femurs, 6 loading conditions and strains measured in 15 anatomical sites) demonstrate the good accuracy of meshless models if compared with experimental measurements (linear regression of predicted vs. measured values: R^2=0.94, slope 0.93, intercept 1.88 $\mu\varepsilon$). The accuracy in predicting superficial strains resulted completely comparable with what obtained by the corresponding FEM models.

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