11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

GEOMETRICAL VALIDITY OF HIGH-ORDER PYRAMIDAL FINITE ELEMENTS

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Key words: *Element Quality, Finite Element Method, Bézier, High-order Method.*

The development of high-order computational methods for solving partial differential equations on unstructured grids has been underway for many years. Such methods critically depend on the availability of high-quality curvilinear meshes, as one badly-shaped element can degrade the solution in the whole domain [2]. The usual way of generating curved meshes is to first generate a straight sided mesh and to curve mesh entities that are classified on the boundaries of the domain. The latter operation introduces a "shape-distortion" that should be controlled if we suppose that the straight sided mesh is composed of well-shaped elements.

In [1] Bergot *et al.* recently proposed a new family of nodal pyramidal finite elements that are conform with hexahedra, tetrahedra and prisms for arbitrary orders. Moreover, they showed that their new pyramidal elements are optimal in the sense that the final error estimate is in $O(hr + 1)$ in L^2-norm [1, 4]. Yet, the characterization of pyramids for which the mapping between the reference and the physical element is invertible remained an open question.

In [3] we recently introduced an algorithm that computes sharp bounds on the Jacobian determinant of triangles, quadrilaterals, hexahedra, tetrahedra and prisms. Those bounds allow to guarantee the geometrical validity of an element, *i.e.* that its mapping is invertible. The key feature of the method is that we can adaptively expand the Jacobian determinant in terms of Bézier functions. Bézier functions have both properties of boundedness and positivity, which allow sharp computation of minimum or maximum of the interpolated functions.

In this work, we propose an adaptation of this method to the pyramids proposed by Bergot *et al.* Instead of classical polynomial Bézier functions, we use functions composed of rational fractions. A good choice of them allows to obtain the necessary properties.

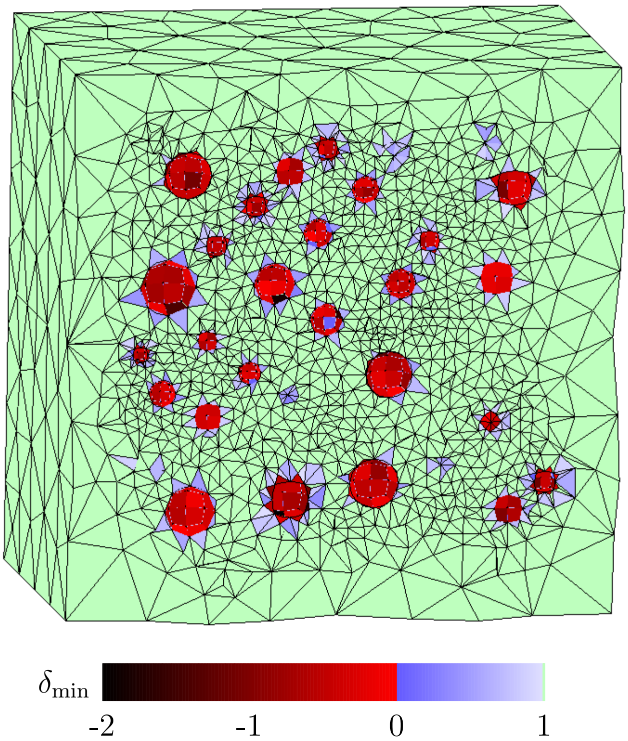

Figure 1: Three-dimensional mesh with second order elements. The geometry consists of a cube with spherical holes. Pyramids make the transition from the hexahedra that fill the holes to the tetrahedra that fill the rest of the volume. Green elements are straight-sided tetrahedra. Blue elements are valid curved tetrahedra or pyramids, while elements between red and black are invalid curved tetrahedra or pyramids.

Results show that we are able to detect all invalid pyramids. On the three-dimensional microstructure of figure 1, second order pyramids have been generated with a naive algorithm. We measured the minimum of distortion $\delta_{\min}$ as defined in [3]. Invalid elements have a negative minimum distortion and we successfully detected all 5589 invalid pyramids (out of 5760) in the grid.

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