11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

GEOMETRY-INDEPENDENT FIELD APPROXIMATION FOR SPLINE-BASED FINITE ELEMENT METHODS

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Key words: *Super-parametric methods, Isogeometric Analysis, Geometry-independent Spline Space, PHT-splines.*

A number of methods have been proposed over the past decade to use geometrical information to build field approximations when solving partial differential equations (PDE): subdivision surfaces [3], implicit surfaces and non-fitted meshes [4], [5] and isogeometric analysis (IGA) [1]. The latter focuses on streamlining the Computer-Aided Design (CAD) to Analysis pipeline. In all cases, geometrical information has to be passed to the numerical approximation used to discretize the field variables.

In IGA, the same spline representation is employed for the geometry and for the field variables solutions. This has the potential to provide closer integration between geometric design and numerical analysis since any change in the CAD model is directly inherited by the field approximation. On the other hand, using the same spline spaces for both geometry and field approximation creates a constraint which may be unwanted, for example when the geometry spline space is not well-suited to approximate the solution of the PDE, in particular when local mesh refinement is required to capture the solution with a tractable computational complexity.

We propose a discretization scheme where the spline spaces used for the geometry and the field variables can be chosen independently. Suppose that the computational domain defined on the parametric domain $\mathcal{P}$ has the following spline representation $\sigma(\mathbf{u}) := \sum_{0\leq i\leq n} \mathbf{c}_i N_i(\mathbf{u})$, where $\mathbf{c}_i \in \mathbb{R}^m, m = 2, 3$, are the control points, $N_i(\mathbf{u})$ are B-spline basis functions for a given knot vector. The key idea of IGA is to represent the solution field $T_I(\mathbf{u})$ with the same spline representation as the computational domain $T_I(\mathbf{u}) = \sum_{0\leq i\leq n} T_i^I N_i(\mathbf{u})$ where T_i^I are the control variables to be solved. Note that when h-refinement and p-refinement are performed, the solution field will have the same parameterization as the refined geometry of the computational domain.

In the proposed method, the solution field can have a different spline representation $T_S(\mathbf{u}) = \sum_{0 \le i \le l} T_i^S M_i(\mathbf{u})$, where T_i^S are the control variables to be solved, and $M_i(\mathbf{u})$ are basis functions spanning the specified spline space defined on the parametric domain $\mathcal{P}$, such as B-splines of different degree, PHT-splines, T-splines and the generalized B-splines. Importantly, the geometry of the computational domain has the same spline representation as the given CAD boundary, which allows the method to preserve the potential to realize seamless integration of CAD and Analysis. The method has the following features: (1) The space of the field variables is independent from that of the geometry of the domain. The method is thus applicable to arbitrary domains with spline representation. (2) It is possible to flexibly choose between different spline spaces with different properties to better represent the solution of the PDE, e.g. the continuity of the solution field. (3) Refinement operations by knot insertion and degree elevation are performed directly on the spline space of the solution field, independently of the spline space of the geometry of the domain, i.e. the parameterization of the given geometry is not altered during the refinement process. Hence, the initial design can be optimized in the subsequent shape optimization stage without constraining the geometry discretization space to conform to the field approximation space.

We present preliminary results based on PHT-spline [2] descriptions for two-dimensional heat-conduction.

Remark: A very similar idea was discussed at the 2013 CISM course on Isogeometric Analysis `http://www.cism.it/courses/C1301/` by Prof. Gernot Beer, and is also known as “Generalised Isogeometric Analysis.”

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