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SOLVABILITY FOR DYNAMIC THERMO–ELASTO–PLASTIC CONTACT PROBLEMS

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The present work aims to give some new mathematical results on existence and uniqueness for thermo–elasto–plastic dynamical contact problems. The situations involving contact abound in industry, especially in engines or transmissions. For this reason a considerable engineering and mathematical literature deals with dynamic and quasi–static frictional contact problems. As a continuation of the work [KrP13] where contact problems are reformulated as partial differential equations with hysteresis operators in the bulk and on the boundary, a full thermomechanical 1D model taking into account the exchange between different types of energy in an oscillating visco–elasto–plastic body in contact with an elasto–plastic obstacle is proposed.

We focus here on an elasto–plastic bar of length L vibrating longitudinally. The bar is free to move on the one end as long as it does not hit a material obstacle, while on the other end a force is applied. The problem for the unknown displacement and temperature is rewritten in accordance with the formalism of hysteresis operators as solution operators of the underlying variational inequalities. Let $u(x,t)$ be the displacement at time t of the material point of spatial coordinate $x \in (0,L)$, and let σ be the σ_{11} component of the stress tensor. The motion is governed by the equation

$$\rho u_{tt} - \sigma_x = 0, \tag{1}$$

where $(\cdot)_x \overset{\text{def}}{=} \dfrac{\partial(\cdot)}{\partial x}$ and $(\cdot)_t \overset{\text{def}}{=} \dfrac{\partial(\cdot)}{\partial t}$, and $\rho > 0$ denotes the mass density. The stress σ is assumed to satisfy the constitutive equation

$$\sigma \overset{\text{def}}{=} \mathcal{P}[\varepsilon] + \nu\varepsilon_t - \beta(\theta - \theta^{\text{ref}}) \quad \text{and} \quad \varepsilon \overset{\text{def}}{=} u_x, \tag{2}$$

where ε is the ε_{11} component of the strain tensor, $\theta(x,t) > 0$ is the absolute temperature which is one of the unknowns of the problem, $\nu > 0$ is the viscosity modulus, $\beta \in \mathbb{R}$ is the thermal expansion coefficient, and $\theta^{\text{ref}} > 0$ is a given referential temperature. The symbol $\mathcal{P}$ denotes a constitutive operator of elasto–plasticity satisfying the following identity:

$$\mathcal{P}[\varepsilon] = \lambda\varepsilon + \mathcal{P}_0[\varepsilon], \tag{3}$$

where $\mathcal{P}_0[\varepsilon]$ corresponds to the plastic stress component with yield point $r > 0$ and elasticity domain $K \stackrel{\text{def}}{=} [-r, r]$ and satisfies

$$\begin{cases} \mathcal{P}_0[\varepsilon](t) \in K & \text{for all } t \in [0,T], \\ \mathcal{P}_0[\varepsilon](0) = \mathrm{Proj}_K(\mathbb{E}\varepsilon(0)), \\ (\mathbb{E}\varepsilon_t(t) - \mathcal{P}_{0,t}[\varepsilon](t))(\mathcal{P}_0[\varepsilon](t) - y) \geq 0 & \text{a.e. for all } y \in K. \end{cases} \tag{4}$$

Here Proj_K is the projection onto K, the constant $\mathbb{E} > 0$ is the elasticity modulus, and $\lambda > 0$ is the kinematic hardening modulus.

Under appropriate regularity assumptions on the data, the existence and uniqueness results for this thermodynamically consistent problem are presented. More precisely, a space discretization is introduced and some a priori estimates are obtained by using both the classical energy estimate and more specific techniques like the Dafermos estimate [Daf82] and some Sobolev interpolation inequalities, see [BIN78, KrP11]. Then the existence result is derived. Concerning the uniqueness result, it follows from the Lipschitz continuity of the nonlinearities.

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