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GALERKIN VARIATIONAL INTEGRATORS FOR SOLID AND FLUIDS MECHANICS

Mattia Penati[*,1], Edie Miglio[2], Nicola Parolini[3], Roberto Porcù[4]

[1] Dip. di Matematica "F. Brioschi" - Politecnico di Milano - `mattia.penati@polimi.it`
[2] Dip. di Matematica "F. Brioschi" - Politecnico di Milano - `edie.miglio@polimi.it`
[3] Dip. di Matematica "F. Brioschi" - Politecnico di Milano - `nicola.parolini@polimi.it`
[4] Dip. di Matematica "F. Brioschi" - Politecnico di Milano - `roberto.porcu@polimi.it`

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The general family of Galerkin variational integrators has been studied and a complete classification of such methods has been proposed. This classification is based upon the type of basis function choosen to approximate the trajectories of material points and the numerical quadrature formula used in time. This kind of numerical technique leads to the definition of arbitrarily high order method in space.

Assuming the validity of some mild hypotheses, which ensure the well posedness of continuous problem, the discrete problem has been studied, proving its well posedness and its approximation properties. Moreover the preserving properties have been extensively studied.

This kind of results are not totally new, some authors studied these methods previously [1, 2, 3, 4, 5]. All of them developed the theory in the context of Hamiltonian mechanics. In the present work a mathematical framework will be developed in order to extend this class of geometric integrators to continuum mechanics.

Different material behaviours (like elasticity and viscosity) as well as global constraints (such as incompressibility) can be casted in this frameword. This class of methods can be used to treat conservative and dissipative processes preserving the geometric structure of the continuous equations and the conservation laws.

The theoretical results are supported by a series of numerical simulation showing the good proporties of the advocated methods. The simulations have been performed using the FEniCS library.

In the context of SOCIS 2013[1] some of these methods have been implemented for the Hamiltonian mechanics problems. In particular the spectral variational integrators will be part of the odepkg Octave package[2]. Some details of the implementation are reported in the project blog[3].

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