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USING TOPOLOGY OPTIMIZATION IN LIGHTWEIGHT DESIGN OF FATIGUE RESISTANT STRUCTURES

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Ease in application design methodology of lightweight and fatigue resistant structures is important in many industrial applications. In this article, a proposal of methodology of the fatigue topology optimization is presented. The Dang Van [1] multiaxial high-cycle fatigue and commonly used von Mises constraints are examined using the constant criterion surface algorithm (CCSA) [2]. The CSSA algorithm does not assume a priori the volume value of optimized structure. The algorithm gives possibility to do optimization with various constraints as well as to solve multi-constraint problems.

The Dang Van hypothesis is based on average stresses in an elementary volume *V*, it considers the average value of shear and normal stresses in this volume. According to Dang Van the fatigue damage appears in a definite time, when the combination of local shear stresses $\tau(t)$ and a hydrostatic stress $\sigma_H(t)$ cuts the borders of an admissible fatigue area.

The numerical convenient form of criterion is as follows [1]:

$$\max_{A} \left[\tau(t) + \kappa\sigma_H(t)\right] \leq \lambda$$

where: *A* is the area of studied object, $\tau(t) = \dfrac{\sigma_1(t) - \sigma_3(t)}{2}$, $\sigma_H(t) = \frac{1}{3}\left(\sigma_1(t) + \sigma_2(t) + \sigma_3(t)\right)$.

The material parameters can be expressed by data from two high-cycle fatigue tests: reversed bending (fatigue limit f_{-1}) and reversed torsion (fatigue limit t_{-1}). For the criterion $\lambda = t_{-1}$, $\kappa = 3\, t_{-1}/f_{-1} - 3/2$.

To show clear differences in influence of the von Mises and Dang Van criteria, the Michell's problem of optimizing truss topology with stress constraints under static and variable load conditions was selected as an optimization example [3] (Fig. 1a-b.). The design domain of numerical example is discretized with 300 × 80 4-node plane stress elements (Young's modulus E = 210 GPa, Poisson's ratio $\nu = 0.3$). The static (F_1 = 100N and F_2 = 200N) and pulsating (amplitude A_1 = 100N and A_2 = 200N, (R = 0)) load conditions were applied in four tests to the chosen structure. In the test, the structures volume are minimized with the von Mises static ($\bar{g}_{vM}$= 205 MPa) and Dang Van fatigue stress constraints ($\bar{g}_{DV}$= 114 MPa (assumed material model (f_{-1} = 190 MPa, t_{-1} = 114 MPa)).

The results of topology optimization are presented in Fig. 1c-f.

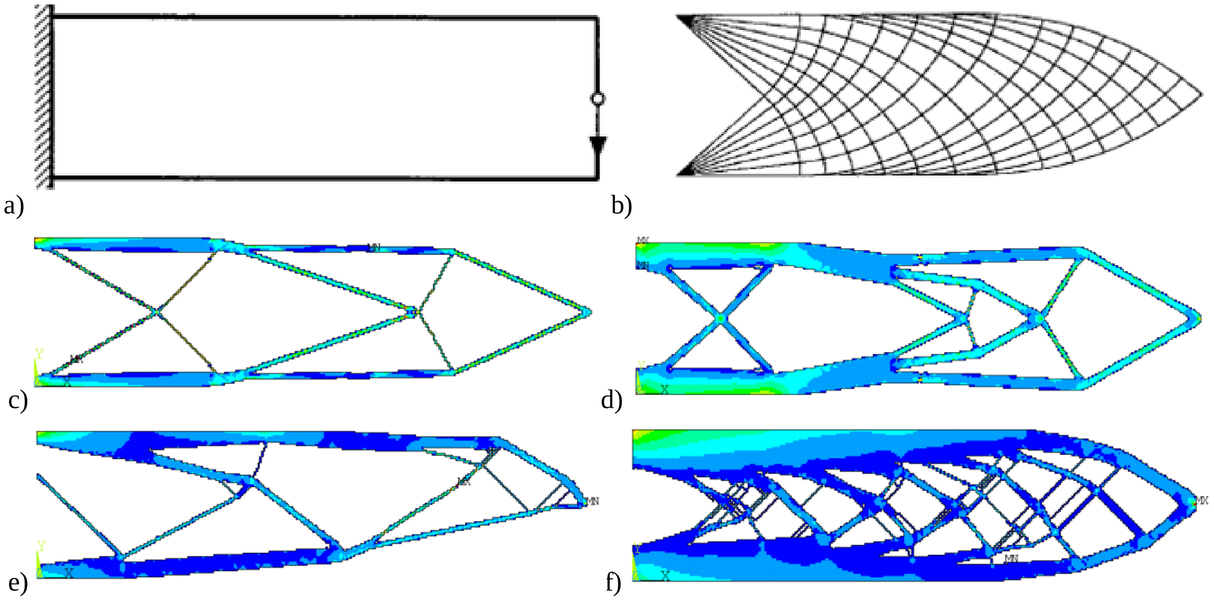

Figure 1: Numerical example: problem description (design domain = 300×80 el. (V_0=100%)) (a), analytical solution (b); numerical solutions for load: F_1=100 N (V_1=15.26 %, σ_{max1} =182 MPa(von Mises)) (c), F_2=200 N (V_2=31.41 %, $\sigma_{max\,2}$ =204 MPa(von Mises)) (d); numerical solutions for pulsating load of amplitude: A_1=100 N (V_3=22.8 %, σ_{max3} =114 MPa(Dang Van)) (e), A_2=200 N(V_4=52.43 %, σ_{max4} =105 MPa(Dang Van)) (f)

One can conclude from presented experiments, that von Mises and Dang Van high-cycle fatigue constraints give different results in optimized structures. It can be clearly seen that the Dang Van criterion discloses the effect of shear stresses in the structure. Moreover, the differences among the volume values (V_1=15.27% < V_3=22.8%, V_2=31.41% < V_4=52.43%) indicate the conservative impact of the Dang Van constraints.

In opinion of the author, the method of topology optimization with multiaxial high-cycle fatigue constraints can be recommended for design of lightweight and fatigue resistant structures.

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