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SURFACE STRESS IN A TWO-COMPONENT COMPOSITE WITH A SLIGHTLY CURVED INTERFACE

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The influence of surface stress on the state of an ideal elastic body, as a rule, is neglected at the macro scale, since it is insignificant compared to the influence of the other forces. However, at the sub-micron scales, quantum and surface effects become dominant in solids, which causes significant changes in the mechanical and electrical properties of the materials. By reduction of the body sizes up to the nanometer scale, the size-effect, that is dependence of bodies' physical properties on geometrical parameters, starts to be experimentally shown [1]. Generally, the size of the interface imperfection and maximum curvature are regarded as a geometrical parameter. Moreover, the surface stress is an additional source of the stress concentration. In our work, using the perturbation method, Gurtin – Murdoch theory of surface elasticity and Goursat – Kolosov complex potentials [2] we reduce the problem to the sequence of hypersingular integral equations. We consider the model of a two-component plane $\Omega_k = \{z: \ (-1)^k \operatorname{Im}(z-\zeta) > 0, \ \operatorname{Re}(z-\zeta) = 0, \ k = 1, 2\}$ with slightly curved interface $\Gamma = \{z: \ z = \zeta \equiv x_1 + i\varepsilon f(x_1)\}$ under the action of unknown surface stress (Duan, 2008). It is supposed that the conditions of the plane strain are satisfied. We assume that the jump of the traction σ^j $(j = 1, 2)$ at the interface is expressed in terms of the surface stress σ^s:

$$\sigma^2(\zeta) - \sigma^1(\zeta) = \Delta\sigma(\zeta) = t^s(\zeta), \tag{1}$$

where $t^s = \dfrac{\sigma^s}{R} - i\dfrac{d\sigma^s}{dx_1}\dfrac{1}{h}$; R is the curvature radius of the interface, h is the metric factor. The constitutive equations for the surface linear elasticity [3] with two elastic parameters similar to the Lame constants and with residual surface stress are used (Duan, 2008). In addition, we suppose that $\sigma_{11}^1 = \sigma \neq 0$, and $\sigma_{22}^k = \sigma_{12}^k = 0$ at infinity.

The solution of the problem is constructed by means of Goursat – Kolosov complex potentials [4]. The condition of a displacement continuity under passing from the volume to the surface leads to the sequence of hypersingular integral equations of the second order in the derivative of the surface stress. For example, the equation for finding the first-order approximation is

$$\sigma_1^{s\prime}(x_1) - \frac{MK_1}{2\pi i}\int_{-\infty}^{+\infty}\frac{i\sigma_1^{s\prime}(t)}{(t-x_1)^2}dt = \frac{MK_2}{2\pi i}\int_{-\infty}^{+\infty}\frac{if'(t)\sigma}{(t-x_1)^2}dt + MK_3 f'''(x_1)\sigma_0^s, \qquad (2)$$

where M, K_1, K_2, K_3 are the parameters depending on the elastic constants of the surface and bulk materials, σ_0^s is the solution from the previous equation. Expanding function $f(x_1)$ in Fourier series, we obtain the solution of the equation (2) in a Fourier series as well.

The influence of the surface stress on the stresses and strains at the interface is analyzed. Stress distribution along the interface is obtained for various stiffness ratio of the materials, elastic parameters and relief forms. In particular, the size-effect is discovered. It becomes apparent in stress dependence on a period of interface roughness if the values of this period have the order of 1-50 nanometers. Moreover, the surface stress has significant influence on the characteristics of a stress-strain state at the interface. For example, if the interface is sufficiently sharp, the normal stress at the boundary is multiplied by 2.7 in comparison with the classical solution. This influence practically disappears when the period equals 100 nanometers or more. Thus, taking into account the surface stress leads to considerable change of stresses at the slightly curved boundary of the half-plane.

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