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A HYBRID, EXPLICIT-IMPLICIT, SECOND ORDER IN SPACE AND TIME TVD SCHEME FOR TWO-DIMENSIONAL COMPRESSIBLE FLOWS

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Explicit schemes are widely used for simulations of unsteady high-speed flows with shock waves. However, in some shock wave flows the time step of explicit time marching becomes severely restricted by particular conditions in a relatively small flow area while the rest of the computational domain admits much higher time steps. There may be different sources of such *temporal stiffness*, e.g. small geometrical features; locally high temperatures and velocities; regions where viscous effects are essential. In this case one may benefit from a hybrid, explicit-implicit, scheme which would turn into the implicit mode in the flow regions causing temporal stiffness while the rest of the flow would be simulated in the explicit mode. A number of such schemes were proposed in the past, with, arguably, the most advanced technique being the hybrid scheme by Men'shov and Nakamura [1]. This scheme (M-N) is of the second order in space but only of the first order of accuracy in time in the fully implicit and hybrid modes and does not belong to the class of TVD (Total Variation Diminishing) schemes. In our recent work [3] we modified the M-N hybridization approach and, inspired by the work [2], introduced a new TVD time limiter for our hybrid scheme via the application of Harten's theorem. These resulted in the hybrid scheme for 1D linear/nonlinear scalar conservation laws, which is a TVD one, of second order accuracy both in space and time in all modes and, as a result, less dissipative than the M-N scheme. The present paper is devoted to the generalization of these developments for adaptive (h-refinement) unstructured triangular grids with node-centered control volumes and the Euler equations. The unstructured grid discretization of the governing equations is solved using the Lower-Upper-Symmetric-Gauss-Seidel (LU-SGS) approximate factorization method [1]. For the Euler equations primitive variables are used for the time-space reconstruction within control volumes. The exact Riemann solver is employed for evaluation of fluxes at control volume interfaces. A node-partitioning procedure is applied to organize the forward/backward sweeps of the LU-SGS procedure.

The new hybrid scheme is applied to a number of test problems chosen to represent the cases with various sources of stiffness mentioned above. Overall, it is demonstrated that

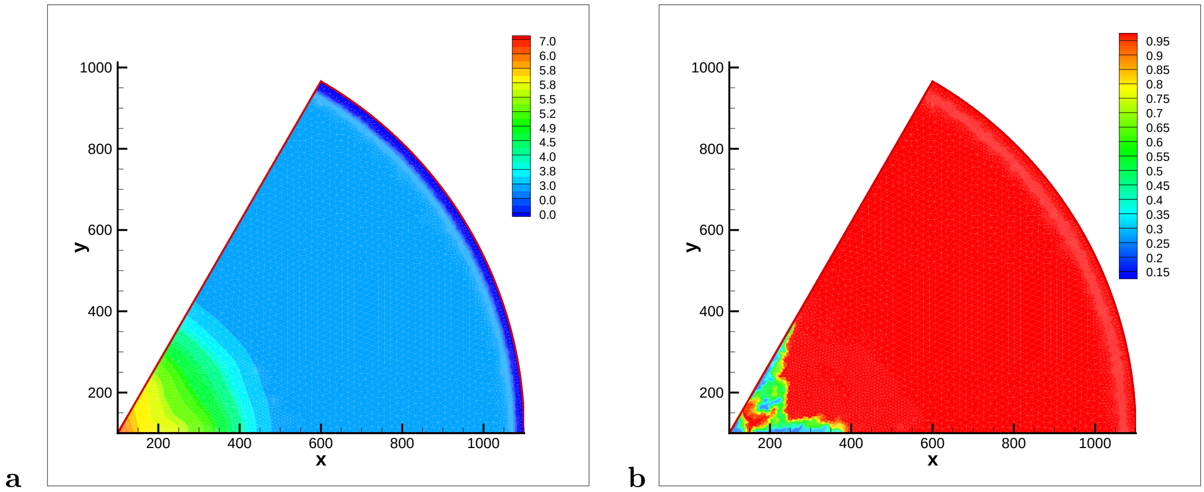

Figure 1: Cylindrical blast wave propagation (only a sector is modeled): (a) Non-dimensional temperature contours (log scale) for the moment when the blast wave is close to the outer cylindrical boundary; the high-temperature core is seen near the explosion center at (100,100); (b) The distribution of the explicitness coefficient ω (for the same time moment), which defines the portion of the explicit mode at each grid node ($\omega = 1$ - fully explicit, $\omega = 0$ - fully implicit). The contact surface is unsymmetrical due to instabilities growing from initial imperfections of the boundary of the energy deposition region.

the proposed hybrid scheme results in the solutions of the same accuracy as the ones produced by its fully explicit mode (the MUSCL-Hancock scheme) while the computational time is significantly reduced. For example, in the simulation of a cylindrical blast wave induced by instantaneous energy deposition within a small cylindrical region the temporal stiffness is due to the high temperature (and hence, the high speed of sound) core remaining at the explosion center after the blast wave has been formed and moved away from the center (see Fig. 1a). As seen from Fig. 1b, the scheme is in its explicit mode in the most part of the computational domain, which includes the blast wave and determines the time step value based on the Courant number $CFL = 0.5$; at the same time it is predominantly implicit closer to the explosion center where the Courant numbers are higher due to high temperature. This results in almost 5 times shorter CPU run time as compared to the same simulation done with the fully explicit method and $CFL = 0.5$ condition (and hence much smaller time step). Simulations of other demonstrative problems (e.g., shock diffraction over a sharp corner, gun tunnel nozzle flow and others) will be presented too.

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