

## STABLE TIME–SCHEME FOR QUASI–INCOMPRESSIBLE TWO–PHASE DIFFUSE–INTERFACE FLOWS

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A main challenge in the simulation of binary–phase flows is the time evolution of interfaces which leads to topological deformations, such as break–up or coalescence. Navier–Stokes/Cahn–Hilliard (NSCH) type equations based on diffuse–interface models naturally overcome the difficulty of capturing the interface by using a diffuse–interface description, the location of which is given by the phase–field variable.

A number of NSCH type equations are available for two–phase flow applications. *Model H* assumes both constituent and the resulting mixture to be incompressible and the model obeys thermodynamics, however it is not straightforward to simulate flows with nonmatched densities [2]. Modified *Model H* is convenient for simulation, but is thermodynamically inconsistent [1]. The quasi–incompressible NSCH [2], on the other hand, can be applied to nonmatched density two–phase flows, meanwhile it dissipates the total energy.

In this contribution, we consider a novel thermodynamically consistent quasi–incompressible NSCH equation. The equation is derived based on the continuum theory of mixtures [3] by the employment of the second law of thermodynamics in the form of free–energy dissipation. We propose a stable time–scheme for the model to satisfy its thermodynamic characteristics at the discrete level. We will also present illustrative numerical results.

### REFERENCES

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