11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A CELL CENTERED FINITE VOLUME SCHEME FOR SOLVING A VECTORIAL DIFFUSION EQUATION ON UNSTRUCTURED GRIDS. APPLICATION TO THE COMPRESSIBLE NAVIER-STOKES EQUATIONS

Pascal Jacq[1,2], Pierre-Henri Maire[2,⋆], Rémi Abgrall[3]

[1] INRIA Bordeaux Sud Ouest France, jacq@inria.fr
[2] CEA-CESTA, 33112 Le Barp France, maire@celia.u-bordeaux1.fr
[3] Universität Zürich Switzerland, remi.abgrall@math.uzh.ch

Key words: *Finite Volume scheme, vectorial diffusion equation, compressible Navier-Stokes equations.*

In this work, we are interested in developing a cell-centered Finite Volume (FV) scheme for solving the compressible Navier-Stokes (NS) equations on two-dimensional unstructured grids, refer to [2]. The inviscid part of the Navier-Stokes equations, *i.e.*, the compressible Euler equations, are discretized employing classical approximate Riemann solvers such as the Roe solver or the HLLC one, refer to [4].

The main novelty of our contribution lies in the development of an original cell-centered FV approximation of the heat-conducting and viscous parts of the NS equations. The heat-conducting part is discretized employing the CCLAD (Cell-Centered LAgrangian Diffusion) scheme described in [3]. Regarding the viscous part, we develop a non trivial extension of the CCLAD approach to the case of a vectorial diffusion equation, which is nothing but the counterpart of the NS momentum equation. Here, the viscous flux is defined by a constitutive law which expresses the deviatoric part of the Cauchy stress in terms of the deviatoric part of the strain rate, *i.e.*, the symmetric part of the velocity gradient tensor. In our FV discretization the degrees of freedom are located at the cell centers. The divergence of the viscous stress tensor is discretized by means of half-edge viscous fluxes which are the projections of the viscous stress tensor onto the unit outward normal to the cell interfaces. It is worth mentioning that the constitutive law defining the viscous stress tensor is not invertible over the space of generic second-order tensors. This lack of invertibility prevents us from applying straightforwardly the CCLAD methodology which consists in employing a local variational formulation to obtain the half-edge viscous flux approximation. To cure this flaw, we follow the seminal idea introduced by Arnold and Falk [1] in the context of linear elasticity. More precisely, it amounts to add a divergence free term to the constitutive law such that it renders it invertible over the space of generic second-order tensors, while keeping the vectorial diffusion equations unchanged. This methodology leads to a robust approximation of the half-edge viscous fluxes in terms of the cell-centered velocity and the half-edge velocities, which are extra degrees of freedom. These auxiliary unknowns are eliminated by solving node-based invertible linear systems which are constructed writting the continuity of the half-edge viscous fluxes across cell interfaces impinging at a given node.

The robustness and the accuracy of this new FV method are assessed against various numerical test cases.

REFERENCES

[1] D.N. Arnold and R.S. Falk "A new mixed formulation for elasticity" *Numer. Math.*, Vol. **53**, 13–30, 1988.

[2] P. Jacq "Finite Volume methods on unstructured grids for solving anisotropic heat transfer and compressible Navier-Stokes equations" *PhD Thesis*, Bordeaux University, 2014.

[3] P.-H. Maire and J. Breil "A high-order finite volume cell-centered scheme for anisotropic diffusion on two-dimensional unstructured grids" *Journal of Computational Physics*, Vol. **224**, 785–823, 2011.

[4] E. Toro "Riemann Solvers and Numerical Methods for Fluid Dynamics: a practical introduction" *Springer*, 2009.