

Time Integration Methods for the Enriched Conformal Decomposition Finite Element Method

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Enriched finite element methods such as the Generalized Finite Element Method (GFEM), the eXtended Finite Element Method (XFEM), and the Conformal Decomposition Finite Element Methods (CDFEM) are powerful tools for multiphase and multimaterial problems. These methods provide discretizations that dynamically adapt to the moving material and phases to accurately capture the interfacial physics and discontinuities.

Time integration in enriched finite element methods is challenging because, as nodes or elements change material or phase, the degrees of freedom change in a discontinuous manner. The degrees of freedom at a node depend on the phases or materials that intersect the elements supported by the node. As these materials move through the domain, the unknowns are created and destroyed in order to accommodate this dynamic discretization. For XFEM, recent work has developed time integration methods that account for the interfacial discontinuities. In [1] time integration methods are developed for handling weak discontinuities in the context of XFEM. More recently, strong discontinuities have also been addressed [2]. Because CDFEM is also an enriched finite element method, there is much in common with these approaches.

The Conformal Decomposition Finite Element Method (CDFEM) is an enriched finite element method that can be used to describe arbitrarily discontinuous physics across dynamic interfaces. A level set is used to describe the location of the moving interface. Nodes are added at the intersection of the level set surface with the edges of the input mesh, and a conforming mesh is generated automatically. Standard unstructured mesh data structures are generated for the resulting conformal mesh in terms of element blocks and side sets. This general framework allows the physics code to describe either weak or strong discontinuities across the interface using standard finite element methods. Completely disparate physics can be employed on either side of the interface.

In this talk, we present a novel method for handling the dynamic discretization produced by enriched finite element methods, specifically CDFEM. The approach is based on moving mesh methods for fixed discretizations, using ideas from Arbitrary Lagrangian Eulerian (ALE) methods. This method is compared to other recent extrapolation-based methods. Versions of the methods are developed that exhibit first and second order accuracy in time. The methods are demonstrated on multiple problems exhibiting both strong and weak discontinuities that vary in time and space, and optimal convergence rates are achieved.

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