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HOPF BIFURCATION WITH 1:2 SPATIAL RESONANCE IN AN AIR-FILLED DIFFERENTIALLY HEATED ROTATING ANNULUS

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We present an analysis of the Hopf bifurcations that occur in a mathematical model that uses the (three-dimensional) Navier-Stokes equations in the Boussinesq approximation to describe the flows of a near unity Prandtl number fluid (i.e. air) in the differentially heated rotating annulus. These bifurcations correspond to the transition from axisymmetric to nonaxisymmetric flow, where the axisymmetric flow loses stability to an azimuthal mode of integer wave number, and rotating waves may be observed. Along this transition, double Hopf (Hopf-Hopf) bifurcations occur, in which there is an interaction of two modes with azimuthal wave numbers differing by one. Of particular interest is the bifurcation involving the waves of wave number m=1 and m=2, i.e. a Hopf bifurcation with 1:2 spatial resonance.

Centre manifold reduction and normal forms are used to predict the behaviour near the transition. However, the axisymmetric solution and its linear stability cannot be found analytically. Therefore, the solutions and the corresponding eigenvalues and eigenfunctions are approximated numerically from the large sparse systems that result from the discretization of the partial differential model equations. In the case of 1:2 spatial resonance, the center manifold equations contain lower-order terms that are not present for the mode-interactions with higher-order resonance. These terms modify the dynamic possibilities near the bifurcation.

The analysis shows that in certain regions in parameter space stable quasiperiodic mixed-azimuthal mode solutions result from the mode-interaction. For the mode-interaction with 1:2 spatial resonance, the analysis predicts that a stable mixed-mode solution arises that is periodic as opposed to quasiperiodic. Comparisons are made with results from laboratory experiments.