

# AN EXTENDED ALGEBRAIC VARIATIONAL MULTISCALE-MULTIGRID-MULTIFRACTAL METHOD (XAVM<sup>4</sup>) FOR LARGE-EDDY SIMULATION OF TURBULENT TWO-PHASE FLOW

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Turbulent two-phase flows are encountered in various industrial applications and natural phenomena. From a computational point of view, turbulent two-phase flows exhibit several particularly demanding challenges such as an adequate representation of the interface between the phases including the incorporation of the corresponding coupling conditions as well as an appropriate approach for turbulence. In this presentation, a novel computational method particularly focusing on the aforementioned issues is proposed for large-eddy simulation of turbulent two-phase flow.

Fixed-grid approaches based on the extended finite element method, which enables the representation of discontinuities in the solution fields by enlarged function spaces, provide a promising framework for numerically simulating complex two-phase flows; see, e.g., [1]. Here, the evolution of the moving interface between the phases is governed by a level-set method, allowing for arbitrary interface deformations. Using Nitsche's method, the continuity of the velocity field across the interface is imposed weakly within the extended finite element method. In particular, face-oriented ghost-penalty terms are imposed on elements intersected by the interface to deal with any cut situation; see, e.g., [2]. Extensions of Nitsche's method in the form of convective flux terms are additionally included to cope with convection-dominated turbulent flows.

In large-eddy simulation of turbulent flow, only the larger flow structures are resolved, while the smaller flow structures, which exhibit a more universal character, are modeled. The multifractal subgrid-scale modeling approach, originally proposed for LES of turbulent incompressible flow in [3], allows for directly closing the nonlinear subgrid-scale terms

arising in the variational multiscale formulation of the incompressible Navier-Stokes equations, as shown in [4]. Based on the driving mechanism of turbulent flow, the subgrid-scale vorticity is first reconstructed by a two-step multifractal process and then inserted into the law of Biot-Savart to calculate the subgrid-scale velocity. The multifractal procedure to approximate the subgrid-scale vorticity consists of a multiplicative cascade to recover its magnitude from the subgrid-scale enstrophy and a decorrelation cascade to determine its orientation. Level-transfer operators from plain aggregation algebraic multigrid methods are applied to further decompose the resolved scales into larger and smaller ones within the multifractal reconstruction process; see also, e.g., [5].

In this talk, the eXtended Algebraic Variational Multiscale-Multigrid-Multifractal Method (XAVM<sup>4</sup>), combining a Nitsche-type extended variational multiscale method for two-phase flow and the multifractal subgrid-scale modeling approach for large-eddy simulation of turbulent incompressible flow, will be derived. To illustrate the potential of the proposed method, results from two numerical examples will be presented. The first example will be selected from various configurations of turbulent bubbly channel flows as considered, e.g., in [6]. A two-phase flow past a backward-facing step will be shown as the second numerical example; see, e.g., [7].

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