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# A FLUX CORRECTED REMAP OF VECTOR FIELDS FOR ALE HYDRODYNAMICS WITH NODAL ELEMENTS

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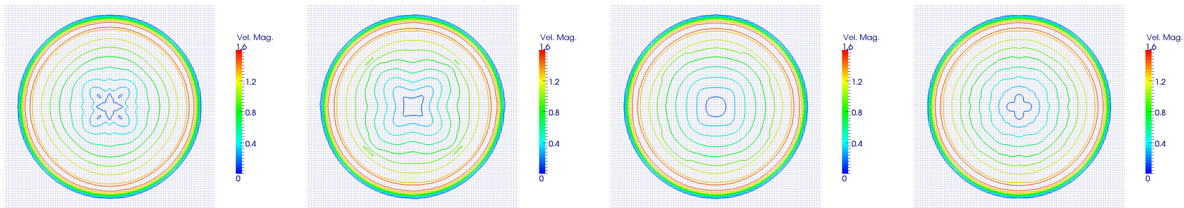
A frame invariant limiting strategy for vector fields is proposed for flux-corrected remap stage of arbitrary Lagrangian-Eulerian (ALE) shock hydrodynamics. During every time step, three stages are performed in the ALE-based shock hydrodynamics: (1) the Lagrangian computation using a variational multiscale (VMS) finite element approach [2]; (2) a mesh smoothing procedure to improve the grid quality; (3) a data remap stage to transfer flow variables from the unsmoothed mesh to the smoothed one. Stage 3 is interpreted as an advection problem with known convection velocity; and it is solved by a second-order finite element method using nodal elements using flux-corrected transport technique to enhance the nonlinear stability.

In previous work [3], the flux-corrected remap applies the FCT strategy to compute flux limiters for scalar fields including both flow density and internal energy (the ***scalar limiter***). This procedure enforces the local extreme diminishing property for these scalar fields during remap, but vector fields like flow velocities are not taken into account. This is a serious issue even for simple problems like the cylindrical symmetric 2D Sedov test: Figure 1(a) shows oscillations in velocity magnitude for solutions by the scalar limiter.

A common practice for limiting vector fields in Eulerian schemes is to apply the scalar limiter to each vector component (the ***componentwise vector limiter***), but this approach is not frame invariant and typically introduces larger dissipations in the directions of coordinate axes (Figure 1(b)). Losing the frame invariance property is a serious issue for Lagrangian computations [4] and thusly a frame invariant limiting strategy is desired for vector fields like flow velocities.

In this work, we propose a frame invariant FCT-based limiting strategy for general vector fields for data remap. The main idea is to first project the vector $\boldsymbol{v}$ at each node onto a certain direction $\boldsymbol{d}$ to obtain a frame invariant scalar field $\boldsymbol{v} \cdot \boldsymbol{d}$, and then apply the FCT




(a) Scalar limiter. (b) Comp. vec. limiter. (c) Vec. limiter I. (d) Vec. limiter II.

**Figure 1**: Contours for velocity magnitude at $T = 0.24$ of 2D Sedov problem.


(a) Scalar limiter. (b) Comp. vec. limiter. (c) Vec. limiter I. (d) Vec. limiter II.

**Figure 2**: Densities at $T = 5.0$ of 2D single material triple point problem.

technique to this scalar field to compute the limiter for all components of $\boldsymbol{v}$. The direction $\boldsymbol{d}$ is chosen such that it predicts the (local) direction in which the oscillations most likely to occur. At present, we consider two different choices: the first one is to use the density gradient $\nabla_{\boldsymbol{x}}\rho$; and the second one is to use the eigenvector associated with the largest eigenvalue of the deviatoric part of $\nabla_{\boldsymbol{x}}\boldsymbol{v}$. Both limiters are frame invariant, and they are labeled ***vector limiter I*** and ***II***, respectively. They lead to significant improvements in the same Sedov test as clearly seen in Figures 1(c) and 1(d).

Another nontrivial problem is the single material triple point test. The solutions for the density by various limiters are provided in Figure 2. The scalar limiter leads to overshoots of density near the slippery contact discontinuities; whereas the three vector limiters are much more stable at these locations. Comparing the three vector limiters, the frame invariant ones provide better resolution around the vortex than the componentwise one without reducing nonlinear stability.

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